# EDUARD BELINSKY, HIS LIFE AND MATHEMATICS

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Eduard S. Belinsky was born in the Ukraine in 1947. He spent most of his "Soviet" life, until 1993 when he moved to Israel with his wife Ella and their two sons Alexei and Vladimir, in Donetsk, a big industrial city and, in a way, the "capital" of the Donetsk coal and metallurgical region.

Luckily, in the year 1965 when Eduard completed his high school education, a university was established in Donetsk. The way it happened is of interest in its own right. There were several higher education institutions in Donetsk in those days, firstly a Polytechnical Institute (M. G. Krein worked for a year in that institute in the 1920s!) and a Medical Institute. There was also a Pedagogical Institute. In 1965, a scientific center of the Academy of Sciences of the Ukraine was established in Donetsk. It consisted of 4 research institutions: an Institute of Applied Mathematics and Mechanics, a Physical-Technical Institute, an Institute of Organic Chemistry, and an Institute of Economics of Industry. A university was established based on the above-mentioned pedagogical institute. Surprisingly, the street where it was located had been given the name University street some 10 years earlier. Crucial for the new university was the fact that the best scientists from the Academic Center became the kernel of its staff, along, of course, with some former staff from the Pedagogical Institute. Here are the names of just a few of the mathematicians who came to Donetsk to create, and work in, that center: Ya. B. Lopatinsky, I. I. Gikhman, G. D. Suvorov, I. I. Danilyuk, P. V. Kharlamov, and A. S. Kosmodamiansky.

Eduard spent five fruitful years at the university with those and other good teachers. His deep interest in mathematics declared itself already in high school, and he started his research activity very early in his student years. His first paper was joint with V. I. Belyi, on approximation in the complex plane, but very soon he switched to work under R. M. Trigub. Trigub came to Donetsk in 1969 and stayed at Donetsk university for the rest of his life. In those years, he paid more attention to Fourier Analysis rather than to "pure" approximation theory. However, problems in Fourier Analysis were studied always in close connection with corresponding important problems of Approximation Theory. Already Eduard's paper submitted in his student years (joint with Trigub and published in Math. Notes in 1974) was impressive. A natural continuation after his graduation in 1970 should have been post-graduate study towards a PhD degree and an academic career. Alas, the situation at that time changed drastically. The Six Day War continued not only for Israelis with everyday terror but also for Soviet Jews. His advisor's attempts to get Eduard accepted as a post-graduate student failed, although Eduard was definitely the best student in his course. Of course, other students, some of them much weaker than Eduard, were given this possibility.

Eduard's decision was to continue his research in his already chosen area with the same advisor, but on an "amateur" basis, and after 8-9 hours per day of working as a computer programmer (there also was a year in the Soviet Army almost immediately after university). Eduard thus had at most

6 hours of sleep, being obliged to get up quite early and spending hours with his mathematics late into the night. The starting point of this life was the summer of 1972.

What was important for Eduard and the next generations of Donetsk mathematicians was the permanent seminar on Approximation Theory and Fourier Analysis, first run by V. I. Belyi and R. M. Trigub, which later, in 1978, split into two seminars under each of the two in accordance with their and their students' interests in either Complex Approximation or in Real Approximation and Fourier Analysis, respectively. The seminars met each week for 2 hours. The tradition was to report on newly obtained results along with more or less detailed proofs. In addition, important and prominent results in the area were given in detail by the participants. From time to time, there were guests at the seminar. It was a difficult problem for Eduard to participate in this seminar while working as a programmer with no flexible working day.

In 5 years, Eduard's PhD thesis was completed, entitled "Application of the Fourier Transform to Summability of Fourier Series". In this work, and in most of the papers published in those years, Eduard continued a systematic study of summability properties of linear means generated by a multiplier function. Such a study in the one-dimensional case was started by Trigub, with emphasis on the close relationship between the properties of these means and the behavior of the Fourier transform of the generating function. Eduard continued that study, concentrating mostly on delicate problems in the multivariate case. His thesis makes for interesting reading even today. Some of the results obtained there were never published, while others have been rediscovered by other mathematicians. The possibilities of publication in the former Soviet Union, especially for one not working in the academic system, were seriously restricted. The main directions of Eduard's research in those years were general theorems on the growth of the Lebesgue constants along with interesting examples, summability of multiple Fourier series at Lebesgue points of two types introduced by Eduard in analogy with the points of weak and strong differentiability, and applications of the results obtained to corresponding problems of approximation theory. Fortunately, there was appropriate council available at the Institute of Applied Mathematics and Mechanics at Donetsk and good people there ready to help to overcome artificial obstacles in difficult situations. (G. D. Suvorov should be mentioned first of all.) Eduard obtained his PhD in 1977. However, that meant nothing for his professional career. He had to continue working at the computational center making time for visiting the seminar and obtaining first rate results late at night. This situation changed only in the late 1980s, in the years of the so-called "perestroika", when Eduard was able to obtain an Associate Professorship at the Donetsk Polytechnical Institute (Technical University today).

In the 1980s, Eduard mostly concentrated on approximation problems inspired by the results of K. I. Babenko from the early 1960s, as well as the works that came next, concerning the problem of sharp orders of $n$-widths and entropy for the class of functions with bounded mixed derivatives. This problem is not yet fully solved. Its hyperbolic nature apparently makes it too complicated. Eduard succeeded in giving sharp estimates for K-functionals of the couple of spaces defined by the Laplace operator and even by a hyperbolic operator. At a certain point, he became interested in problems of approximation of functions by trigonometric polynomials with a given number of harmonics, including estimates of trigonometric $n$-widths. Some of his papers on this subject certainly were of a pioneering nature. It had soon become clear to Eduard that only probabilistic methods could bring essential progress in this topic.

As mentioned, in 1993 Eduard and his family decided to move to Israel, where he continued to work on the problems mentioned. In parallel, a survey on the so-called $A^*$ algebra was prepared in that period jointly with Liflyand and Trigub. This was what he presented at his first conference abroad, the ICM94 in Zürich. Unfortunately, not finding a proper position in Israel, he moved to Harare, Zimbabwe, in 1996, and then in 2000 got a professorship in Barbados. Both places were not rich mathematically, and good students were very much an exception.

Eduard was very honest both in mathematics and everyday life. He was very devoted to his family, was also ready to contribute as much of his time as needed to his students. Those who expressed even a slight wish to challenge themselves in mathematics could have his full attention and help.

Let me mention one of the peculiarities of Eduard as a person and a mathematician. Being fully concentrated on a problem, he was able to become inspired by a problem of a completely different nature, to successfully attack it, and then return to his main interest with in no way losing the thread of the study. Not many of us are able to "jump" that quickly and that successfully from problem to problem.

Deep penetrating methods and problems in probability naturally led Eduard to contributing to the so-called small ball problem by combining his knowledge and experience in both approximation and probability.

The last year of his life that he was able to work on mathematics, 2003, was extremely fruitful. Almost the entire year after that, 2004, passed in a hopeless struggle with cancer.

Many of Eduard's results were presented by him and Trigub in their joint monograph "Fourier Analysis and Approximation of Functions", Kluwer-Springer, 2004. A newly printed copy of this book was one of the last impressions in his life before his untimely passing.

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