

# The life and mathematics of Géza Grünwald

as told by

[Paul Turán](#)

at the April 1, 1955, meeting of the Bolyai Mathematics Society in Budapest

[as partially translated by [Szilárd Révész](#) and as rewritten, excerpted, commented, and hopelessly ruined by [P.N.](#)]

First I started to write about Géza Grünwald in 1943, a little after I found out he had died. It was hard to believe that it was at the April, 1942, meeting of the Bolyai Society when we last met. At the end of the meeting Géza mentioned that he was called up for active military duty in labor service.

Young people are shocked when facing death, they can hardly believe in it. I felt just like that and kept hoping that there must have been an error or a misunderstanding, and, in the end, I stopped writing.

During the liberation of Budapest in 1945, we all became personally acquainted with death. As time went by, the facts started to come out. We learned that the labor service unit he served in was in fact a "death unit" made up of completely innocent "politically unreliable" people as punishment for an act of sabotage in the city of Győr [the birth place of the Riesz brothers - by P.N.]. Within a few months the entire unit was executed except for five lucky survivors, among whom was Mr. István Kossa [a well known politician in Hungary in the 1950-60 era whose book entitled "From the Danube to the Don", gives a detailed account of the lives and death of both the victims and the masters in this labor service battalion] who was a good friend of Grünwald and who stayed with him until his execution.

According to Kossa and the official paperwork, Grünwald died on September 7, 1943, at the age of 31. I had nightmares when reading and remembering the obituary to the extent that I could hardly suppress them as time went by. I will not elaborate on the details [I wish he had - by P.N.]. Let me just say that much that all those who are responsible for his death deserve contempt [most of them were actually executed right after WWII - by P.N.].

Géza Grünwald was born in Budapest on October 18, 1910. His father was a house painter who supported [or at least attempted to support] both him and his brother Gyula. Grünwald attended the same high school as Paul Erdős whom he soon befriended. They used to walk a great deal in the city park in Budapest [Városliget] where they spent a great deal of time on trying to outdo each other in mental arithmetics and trying to beat each other in chess. Grünwald had special chess talents. During his subsequent years at the University of Szeged, he even beat the current chess champion of the city.

Erdős' father, Lajos, was teaching mathematics in the same high school. He soon discovered Grünwald's talent and started to help him both spiritually and financially. The latter, in view of Grünwald's less than modest financial background, was quite essential.

In 1927, Grünwald contracted tuberculosis, and it was Lajos Erdős who helped him to receive treatment in a sanatorium where he spent an entire year.

Because of his illness, he passed his final exams and graduated from high school only in 1929, without any special honors.

Not having been admitted to any Hungarian university, he tried to study in Italy while having major financial difficulties.

It was again Lajos Erdős who drew Alfréd Haar's attention to Grünwald. Haar invited him to an interview which was successful and he was accepted into the University of Szeged's mathematics program.

Starting with his sophomore year, for four consecutive years he was awarded a university prize in mathematics research. During his senior year the award was given for his work on interpolation which became the basis of his doctoral dissertation. He defended the latter on December 4, 1935.

It may be interesting to point out that he shared his senior year mathematics award with Béla Szökefalvi-Nagy.

Haar died on March 16, 1933, and that year Grünwald used the code name "Haar" for his paper submitted to the student competition. The code names in the subsequent years were "Beta", "Gamma", and "Delta", respectively.

In September of 1936, he was awarded a degree in mathematics and physics education.

Right after that he completed his military training and took part in several war exercises. As of 1941, he was "demoted" to "unarmed labor service" eligibility [he was Jewish, that is, an "enemy of the people" - by P.N.].

He married his schoolmate, Anna Szilágyi in 1938. They produced one child, Éva.

In September of 1937 he was hired by Egyesült Izzó [nowadays it is a subsidiary of General Electric - by P.N.] as a research mathematician to work with Zoltán Bay.

In addition to some actuaries, Grünwald must have been one of the first industrial mathematicians.

It is tragic that precisely this job which promised to give him the hope of a stable life became the catalyst for his sufferings and eventual murder.

Grünwald's mathematical interests were wide-ranging. He used to talk about his experiences in Szeged to his friends in Budapest with much enthusiasm. Nevertheless, his primary research area, that is, approximation theory, had no Szegedian roots [well, one might successfully argue that Haar, Kalmár, Riesz, Sz.-Nagy, Vincze, just to name a few, did indeed work, at least to some extent, in approximation theory - by P.N.]. Approximation Theory [with capital "A" and "T"] was largely initiated by Sergei Natanovich Bernstein. Our group of young researchers in Budapest learned of and studied approximation theory through the works of Leopold Fejér. Let me mention that only Tibor Gallai and I survived the Holocaust, the rest of our group perished. Grünwald regularly attended our weekly meetings which we held near the statue of Anonymous in the city park. There was no equivalent opportunity in Szeged so that it was natural that Grünwald became interested in issues discussed in our group.

[from now on Turán's words are interspersed with P.N.'s comments]

Then Turán goes on to explain how they were discussing ideas stemming from the Banach-Steinhaus theorem [the uniform boundedness principle] and how they were trying to prove and/or disprove convergence of Lagrange interpolation on countable sets, and/or almost everywhere and/or everywhere. This is directly related to similar problems in trigonometric Fourier series where, due to the works of A. N. Kolmogorov, Lennart Carleson, and many others,

the convergence/divergence problems have been almost completely settled. My [P.N.'s] favorite is the Carleson-Kahane-Katznelson theorem according to which for every set of Lebesgue measure zero there is a continuous function whose trigonometric Fourier series diverges on this set. As far as I know, it is still an open problem whether this set could be the precise set of divergence.

As we all know well, independently of Grünwald and simultaneously with Grünwald, Józef Marcinkiewicz has settled the same problems [it seems that, for a change, Marcinkiewicz was murdered by the Soviets along with many other Polish prisoners of war - by P.N.].

They both proved that Lagrange interpolation based at the zeros of Chebyshev polynomials can diverge everywhere even for continuous functions. This is certainly Grünwald's best result and one of the best for Marcinkiewicz as well. It is safe to say that this result is a "classic" although these days occasionally much lesser results are also referred to with similar adjectives [says P.N.]

Turán prefers Grünwald's approach. I [P.N.] disagree. Grünwald's proofs "work" whereas Marcinkiewicz's proofs helped to create new areas in mathematics subsequently perfected by [his adviser] Antoni Zygmund and the group of incredibly talented people who consider Zygmund their mathematical ancestor [Zygmund told me that Marcinkiewicz was the strongest mathematician he ever met - I wonder if I am making this up or he told this to others as well - by P.N.].

Then Turán elaborates on Grünwald's other results which in my opinion were moderately interesting. Of course, I [P.N.] have the unfair advantage of looking at Turán's exposition almost 50 years after it had been written and during this period approximation theory has drastically changed its primary foci and it has taken numerous unexpected turns.

Then Turán raises the natural question as to Grünwald's future and potential impact in approximation theory had he not been murdered by his compatriots [my words - P.N.]. He points out that Grünwald was a communist who occasionally talked about his ideas to his friends. Turán mentions that he and his friends first heard of dialectic materialism from Grünwald during the regular Sunday hiking and rowing tours.

We can only speculate whether Turán is telling the truth here or just playing the game [the article was written in 1955 during the still Stalinist era]. I could never even find out whether my own father, a schoolmate of Turán and George Mikes, and a classmate of [Robert Capa](#), was a communist in the 1930's or not. I think the answer depends on whom you ask and when you ask.

Then Turán talks about the series of lectures by Grünwald on ergodic theorems in 1937 or 1938.

Finally, Turán points out that for Grünwald mathematics was not a job but a way of life [surprise surprise - by P.N.]. He dealt with both major and minor problems, even loved to solve problems published in *Jahresbericht der Deutschen Mathematiker-Vereinigung*. Interestingly, he never worked for *Középiskolai Matematikai Lapok* although this could be explained by the fact that he had not decided what profession to pursue until his graduation from high school.

The article finishes with a few emotional sentences which, in 2003, sound a little peculiar. The bottom line, in my words, is that Grünwald, his mother, and his brother, plus 5,999,997 more were murdered and we should neither forgive nor forget.

I, P.N., was born after the Holocaust and never met most of the "famous" Hungarian mathematicians of the first half of the twentieth century. However, I was lucky enough to have met and known Paul Turán. He was, and still is, one of my mathematical heroes and I still miss him even though he died more than 25 years ago [September 26, 1976].

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