

**Jubilee date**

## **Vitalii Pavlovych Motornyi (on occasion of the 80th birthday)**

July 28, 2020 marked the 80th anniversary of the birth of Vitalii Pavlovych Motornyi, Corresponding Member of National Academy of Sciences of Ukraine.



V. P. Motornyi was born in Melitopol in the family of a serviceman. His father, Pavlo Petrovych, died in 1941 during the Defense of Sevastopol.

In 1957, V. P. Motornyi began his career as a mechanic at the Dnepropetrovsk Locomotive Factory. In 1958 he entered the Physics Department of the Physics and Mathematics Faculty of Dnepropetrovsk State University (now — Oles Honchar Dnipro National University). Attending classes of such teachers of Mathematics as Ye. M. Kilberg, Yu. F. Leshchinsky, and then a mathematical circle led by O. P. Timan, Yu. A. Brudny, B. D. Kotlyar, Vitalii Pavlovych realized that his future is connected with Mathematics. He moved to Mathematics Department, which he graduated in 1963 and received a diploma with honors in the specialty “Mathematics”.

V. P. Motornyi finished postgraduate studies in this specialty in 1966 and defended Ph.D. thesis on the topic “Approximation of functions by algebraic polynomials in the space  $L_p$ ” at the specialized council of Dnepropetrovsk State University in 1967.

During postgraduate studies, V. P. Motornyi started pedagogical activity at Dnepropetrovsk University, where until 1974 he worked as an Assistant, Senior Lecturer, Associate Professor of the Department of Functions Theory, which at that time was headed by Mykola Pavlovych Korneichuk. In 1974, Vitalii Pavlovych became the Head of the Department of Functions Theory of Dnepropetrovsk State University and worked in this position until the reorganization of the department in 2010. Now he is Professor at the Department of Mathematical Analysis and Functions Theory. From 1977 to 1980 V. P. Motornyi was the dean of the Faculty of Mechanics and Mathematics of DSU, and from 2007 to 2014 he worked part-time as a Head of the laboratory “Optimization of approximation by polynomials and splines” of the Institute of Applied Mathematics and Mechanics of NAS of Ukraine.

In 1975 V. P. Motornyi successfully defended his doctoral thesis on the topic “Extremal problems of quadratures theory and approximation of functions” at V. O. Steklov Institute of Mathematics of the USSR Academy of Sciences, and in 1977 he was awarded the academic title of Professor.

In his memoirs about Dnepropetrovsk (now the city of Dnipro), academician S. M. Nikolsky named V. P. Motornyi “a major member of Approximation Theory”. In this area, Vitalii Pavlovych obtained a number of fundamental results that brought him worldwide recognition and continue to have a great impact on the development of further researches of many mathematicians nowadays. His scientific achievements are reflected in more than 140 scientific publications, including 2 monographs. Among his students there are 13 Ph.D.’s, two defended their doctoral theses.

In 1991 V. P. Motornyi was awarded the title “Honored Worker of Science and Technology” of the Ukrainian SSR, and in 1994 his scientific activity was recognized with the State Prize of Ukraine in the field of Science and Technology. V. P. Motornyi was elected a Corresponding Member of National Academy of Sciences of Ukraine for the Department of Mathematics in 2000 and became Laureate of the M. O. Lavrentiev Prize of National Academy of Sciences of Ukraine in 2010.

For numerous times, V. P. Motornyi has presented Soviet and Ukrainian Mathematics at international mathematical congresses and conferences. He lectured at S. Banach International Mathematical Center in Warsaw and on a specialized term on Approximation Theory in Haifa, and in various mathematical schools. For many years, he had been the Chairman of the specialized Council for Ph.D. defense at Dnepropetrovsk University, a member of the specialized Council for the defense of doctoral theses at Institute of Applied Mathematics and Mechanics of NAS of Ukraine. For more than 30 years, he was the permanent executive editor of scientific compendium in Mathematics of Dnipropetrovsk University, and now he is executive editor of the scientific journal “Researches in Mathematics”. Also, V. P. Motornyi is a member of the editorial boards of leading scientific mathematical journals “Ukrainian Mathematical Journal” and “Ukrainian Mathematical Bulletin”, published in Ukraine. Vitalii Pavlovych has been leading an authoritative inter-university research seminar on Functions Theory among specialists for many years. In 2015, Oles Honchar Dnipropetrovsk National

University together with Institute of Mathematics of National Academy of Sciences of Ukraine and Taras Shevchenko Kyiv National University held International Scientific Conference “Approximation Theory and its Applications” in Dnipropetrovsk on the occasion of the 75th anniversary of V. P. Motornyi.

For his substantial contribution to the development of the Faculty of Mechanics and Mathematics and the development of mathematical education at the university, V. P. Motornyi was awarded the title “Honored Professor of Dnipropetrovsk University”. He lectures a number of fundamental courses for Mathematics students, was recognized as the best lecturer at Dnipropetrovsk University, was awarded the badge of the USSR Ministry of Education “For excellent successes in work”, was the Chairman of section of All-Union competition for the best student scientific work, Chairman of the Jury of Republican round of the Olympiad “Student and scientific and technical progress”. For great and tireless work with schoolchildren and teachers of Dnipropetrovsk region, he has repeatedly been awarded with the badge “Excellence in Public Education” and diplomas of Ministry of Education.

For many years V. P. Motornyi was the Chairman of the Jury of Regional Olympiads for young mathematicians, and since 2015 — the honorary Chairman of the Jury of the IV round of All-Ukrainian Olympiad for Schoolchildren in Mathematics. For a long time, he taught at Dnipropetrovsk Regional Institute of Postgraduate Pedagogical Education (now — Communal institution of higher education “Dnipro Academy of Continuing Education” of Dnipropetrovsk Regional Council).

In his scientific work, V. P. Motornyi was always focused on solving challenging problems of fundamental importance. Having defended successfully his Ph.D. thesis in 1967, V. P. Motornyi’s research interests were concentrated on several directions: 1) approximation of classes of functions by algebraic polynomials taking into account the position of a point on the interval, 2) convergence of Fourier series in Jacobi polynomials in the spaces  $L_p$ , 3) optimal quadrature formulas on classes of functions, 4) extremal properties of splines and widths of classes of functions; 5) approximation of periodic functions by trigonometric polynomials in the mean. Already by 1973, V. P. Motornyi had obtained fundamental results in each of these directions, that provided solutions to difficult problems in Approximation Theory and formed the basis of his doctoral thesis. Let us present a brief description of main results of this period.

One of the most important problems in Approximation Theory is constructive characterization of classes of functions. For non-periodic functions, this problem remained open for a long time. Its solution became possible only thanks to the work of S. M. Nikolsky in 1946, in which he proved the possibility of approximating functions  $f \in W_\infty^1[-1, 1]$  by algebraic polynomials with improved approximation near the ends of the interval and at the same time asymptotically the best on the whole class. Soon afterwards, in the works of O. P. Timan and V. K. Dzyadyk, the problem of constructive characterization of the Hölder classes  $H_C^{r+\alpha}$ ,  $r = 0, 1, 2, \dots$ ,  $\alpha \in (0, 1)$ , in the space  $C_{[-1,1]}$  was solved. It was established that  $f \in H_C^{r+\alpha}$  if and only if there exists a sequence of algebraic polynomials  $P_n$  such that

$$\left\| \frac{f(x) - P_n(x)}{(\sqrt{1-x^2} + 1/n)^{r+\alpha}} \right\|_{C_{[-1,1]}} = O\left(\frac{1}{n^{r+\alpha}}\right). \quad (1)$$

These works influenced the appearance of a hypothesis that in the spaces  $L_p[-1, 1]$  an equality similar to (1) provides a constructive characterization of the Hölder classes  $H_p^{r+\alpha}$ . Therefore, the result obtained by V. P. Motornyi in 1966 and published in 1967 in the Proceedings of the USSR Academy of Sciences was quite unexpected. In this paper, in particular, Jackson's theorem was strengthened in the space  $L_p[-1, 1]$ . Namely, it was proved that for every function  $f \in H_p^{r+\alpha}$ ,  $r = 0, 1, 2, \dots$ ,  $\alpha \in (0, 1)$ , there exists a sequence of algebraic polynomials  $P_n$  such that the relation holds

$$\left\| \frac{f(x) - P_n(x)}{(\sqrt{1 - x^2} + 1/n)^{r+\alpha}} \right\|_{L_p[-1,1]} = O\left(\frac{\ln^{1/p} n}{n^{r+\alpha}}\right). \quad (2)$$

Furthermore, it is impossible to remove the term  $\ln^{1/p} n$  from the right-hand side of (2) for all functions  $f \in H_p^{r+\alpha}$ . This remarkable result, however, did not provide a constructive characterization of the classes  $H_p^{r+\alpha}$ . V. P. Motornyi constructed a new method of approximation in the case  $r = 0$  and, in terms of this method, presented a constructive characterization of the class  $H_p^\omega$ .

The results of investigations on approximation of functions by algebraic polynomials in the spaces  $L_p$  turned out to be useful in the study of the convergence of the Fourier-Legendre series in these spaces. V. P. Motornyi investigated the conditions for the convergence of these series in the spaces  $L_p$  providing that the Lebesgue constants are unbounded ( $1 \leq p \leq 4/3$ ,  $4 \leq p < \infty$ ). He established an interesting qualitative fact that in case  $p \in (1, 4/3]$  with the improvement of differential-difference properties of the function, the growth of the Lebesgue constants has less effect on the convergence. Moreover, for functions with sufficiently good differential-difference properties the Fourier-Legendre sums in the spaces  $L_p$  ( $1 < p \leq 4/3$ ) provide an approximation that has an order not worse than the order of the best approximation. These results constituted one of the chapters of V. P. Motornyi's doctoral thesis, and researches in this direction continue in the works of his students.

Another chapter of V. P. Motornyi's doctoral thesis is devoted to the problems of approximation of classes  $W_\alpha^r H_X^\omega$  of convolutions of periodic functions  $\varphi \in H_X^\omega$ , having zero mean over the period, with the kernel

$$K_\alpha^r = \sum_{k=1}^{\infty} k^{-r} \cos\left(kt + \frac{\pi\alpha}{2}\right), \quad r > 0, \quad \alpha \in \mathbf{R},$$

by arbitrary summation methods of the Fourier series  $u_n(x; \lambda; t)$ , defined by the triangular matrix  $\lambda$ . For the upper bounds of deviation

$$\mathcal{E}_n(W_\alpha^r H_X^\omega; \lambda) := \sup \{ \|x - u_n(x; \lambda; \cdot)\|_X : x \in W_\alpha^r H_X^\omega \},$$

V. P. Motornyi obtained very general inequalities

$$\mathcal{E}_n(W_\alpha^r H_X^\omega; \lambda)_L \leq \mathcal{E}_n(W_\alpha^r H_C^\omega; \lambda)_C, \quad (3)$$

generalizing the well-known S. M. Nikolsky's relations. Also, he indicated cases when above inequalities turn into equalities or asymptotic equalities. In particular, equality

in (3) holds for arbitrary positive summation methods if  $\omega(t) = t$  and  $\alpha = r$ ,  $r = 0, 1, 2, \dots$  or  $\alpha = r - 1$ ,  $r = 1, 2, 3, \dots$ . If  $\omega$  is a concave modulus of continuity then the equality in (3) holds for arbitrary positive summation methods in the cases  $\alpha = r$ ,  $r = 1, 3, 5, \dots$  or  $\alpha = r - 1$ ,  $r = 2, 4, 6, \dots$ .

In 1973 V. P. Motornyi solved A. M. Kolmogorov's problem on the best quadrature formula of the form  $\sum_{k=1}^n \rho_k f(x_k)$  on the most important functional classes ( $W_C^r$ ,  $r > 3$ ,  $W^r H^\omega$ , where  $\omega$  is a concave modulus of continuity and  $r$  is odd, and  $W_L^r$ ,  $r = 4, 6, \dots$ ) of  $2\pi$ -periodic functions. This result was central to his doctoral thesis. For the classes  $H^\omega$ , the optimal quadrature formula was found earlier by M. P. Korneichuk, who used a very simple idea. First, the error

$$R_n = \sup_{f \in H^\omega} \left\{ \int_0^{2\pi} f(x) dx - \sum_{k=1}^n \rho_k f(x_k) \right\}$$

of the rectangle formula ( $x_k = \frac{2\pi k}{n}$ ,  $r = 0, 1, \dots, n - 1$ ,  $\rho_k = \frac{2\pi}{n}$ ) on the class  $H^\omega$  was calculated. Then, for every system of points  $X = \{x_k\}$ , a non-negative function  $f_X \in H^\omega$  was constructed such that it vanishes at the points  $x_k$  and satisfies inequality  $\int_0^{2\pi} f_X(t) dt \geq R_n$ . This immediately implies that the rectangle formula is optimal on the considered functional class. This technique allowed subsequently to solve the problem of the best quadrature formula on the classes  $W^1 H^\omega$ . However, for the class  $W_C^3$ , the construction of the mentioned function  $f_X$  was fraught with significant technical difficulties. Instead of explicitly constructing a non-negative function, V. P. Motornyi proved, using topological methods, the existence of an  $\omega$ -spline vanishing at a given system of points. This became one of the key factors that made it possible to solve the problem of the best quadrature formula on the mentioned classes of functions. To prove inequality  $\int_0^{2\pi} f_X(t) dt \geq R_n$ , V. P. Motornyi proposed a new method that allowed establishing lower estimates for some numerical characteristics of perfect  $\omega$ -splines with the help of Korneichuk's  $\Sigma$ -rearrangement apparatus.

The methods and results of the work on quadrature formulas found applications in solving other important problems of Approximation Theory, in particular, the problems of finding widths and of optimal recovery of functions and functionals. Soon after the publication of this work by V. P. Motornyi, his joint work with V. I. Ruban appeared, in which the Kolmogorov widths  $d_{2n-1}(W^r H^\omega, L)$ , where  $\omega$  is a concave modulus of continuity, and a lower bound for the Gelfand widths  $d^{2n-1}(W^r H^\omega, L)$  were found.

In 1987, Vitalii Pavlovych again turned his attention to the problem of optimal quadrature formulas on the classes  $W^r H^\omega$  ( $\omega$  is a concave modulus of continuity), proving in joint work with A. O. Kushch that the rectangle formula is the best quadrature formula on the class  $W^2 H^\omega$ . And in 1998, a virtuoso mastery of the apparatus of  $\Sigma$ -rearrangements allowed Vitalii Pavlovych to solve the problem of the best interval quadrature formula on the classes  $W_\infty^r$ ,  $r = 2, 3, \dots$ .

Although, having defended his doctoral thesis, V. P. Motornyi repeatedly returned to each of the above directions, nevertheless, the focus of attention remained on the approximation of functions by algebraic polynomials, taking into account the position of a point on the interval (both in uniform and integral metrics). After S. M. Nikolsky's work in 1946, this topic became the focus of attention of many researchers. In particular,

O. P. Timan generalized S. M. Nikolsky's estimate to the case of approximation of functions  $f \in W_\infty^r$  of arbitrary smoothness. V. M. Temlyakov for  $r = 1$  and R. M. Trigub for all positive integer  $r$ 's improved the estimate of the remainder in O. P. Timan's result by proving that, for every function  $f \in W_\infty^r$  and  $n \geq r - 1$ , one can specify an algebraic polynomial  $P_{n,r}(x)$  of degree at most  $n$  such that for all  $x \in [-1, 1]$  the inequality holds

$$|f(x) - P_{n,r}(x)| \leq \frac{K_r}{n^r} \left( \sqrt{1 - x^2} \right)^r + C_r \frac{(\sqrt{1 - x^2})^{r-1}}{n^{r+1}}, \quad (4)$$

where the constant  $C_r$  depends only on  $r$ . In 1999, V. P. Motornyi obtained generalizations of estimate (4) to the case of arbitrary  $r > 0$ . V. P. Motornyi also obtained similar estimates for functions in classes  $W^r H^\omega$  and for some classes of singular integrals. In 2001 V. P. Motornyi proved that, for every function  $f \in W^r H^\omega$ , there exists a sequence of algebraic polynomials  $P_{n,r}(x)$  of degree at most  $n \geq r$  (for  $r = 0, n \geq 2$ ) such that

$$\begin{aligned} |f(x) - P_{n,r}(x)| &\leq \frac{K_r}{2} \left( \frac{\sqrt{1 - x^2}}{n} \right)^r \omega \left( \frac{2K_{r+1}}{nK_r} \sqrt{1 - x^2} \right) + \\ &+ C_r \frac{(\sqrt{1 - x^2} + \frac{1}{n})^{r-1} \omega \left( \frac{\sqrt{1 - x^2}}{n} + \frac{1}{n^2} \right) \ln n}{n^{r+1}}, \end{aligned} \quad (5)$$

where  $\omega(t)$  is a concave modulus of continuity possessing property:  $t\omega'(t)$  does not decrease, and the constant  $C_r$  depends only on  $r$ . Later, V. P. Motornyi obtained another form of this estimate, which differs from (5) in the form of the remainder that converges to zero faster near the ends of the segment. In 1995 V. P. Motornyi, in collaboration with O. V. Motorna, obtained the asymptotic estimate

$$E_n (W_p^r)_1 = \left( \frac{1}{2\pi} \int_{-1}^1 \left( \sqrt{1 - x^2} \right)^{rq} dx \right)^{\frac{1}{q}} \|\varphi_{n,r}\|_q + o \left( \frac{1}{n^r} \right), \quad \frac{1}{p} + \frac{1}{q} = 1, \quad (6)$$

for arbitrary positive integer  $r$  and  $p \in (1, \infty]$ , where  $\varphi_{n,r}$  is the perfect Euler spline, and the norm  $\|\varphi_{n,r}\|_q$  is calculated in the space  $L_q[-\pi, \pi]$  of  $2\pi$ -periodic functions. Estimate (6) interpolates the results obtained earlier by S. M. Nikolsky for  $p = 1$  and O. V. Motorna for  $p = \infty$ . In another work in 1995 V. P. Motornyi and O. V. Motorna established the asymptotics of the best approximation by algebraic polynomials in the mean of the classes  $W^r H^\alpha$ :

$$E_n (W^r H^\alpha)_1 = \frac{1}{2\pi} \int_{-1}^1 (1 - t^2)^{\frac{r+\alpha}{2}} dt \cdot \|f_{n,r,\alpha}\|_1 + o \left( \frac{1}{n^{r+\alpha}} \right),$$

where  $f_{n,r,\alpha}$  is the  $r$ -th periodic integral having zero mean of  $2\pi/n$ -periodic odd function  $f_{n,0,\alpha}$  defined on  $[0, \pi/n]$  by the formula

$$f_{n,0,\alpha}(t) = \begin{cases} 2^{\alpha-1} t^\alpha, & 0 \leq t < \frac{\pi}{2n}, \\ 2^{\alpha-1} \left( \frac{\pi}{n} - t \right)^\alpha, & \frac{\pi}{2n} \leq t \leq \frac{\pi}{n}. \end{cases}$$

In addition, V.P. Motornyi and O.V. Motorna, in collaboration with P.K. Nitiema, in a series of works from 1996 to 1999 solved the problem of asymptotically sharp estimation of quantities  $E_n(W_1^r)_1$  for arbitrary  $r > 0$ .

In recent years, a number of papers by V.P. Motornyi on the problem of the best one-sided approximations of functions by algebraic polynomials have appeared. Particularly, in the joint paper of V.P. Motornyi and A.M. Pasko in 2004, an asymptotically sharp estimate was obtained for the best one-sided approximation of the classes  $W_1^r$ :

$$E_n^\pm(W_1^r)_1 = \frac{2}{n^r} \cdot \sup_t |B_r(t)| + O\left(\frac{1}{n^{r+1}}\right), \quad (7)$$

where  $B_r(t)$  is the Bernoulli kernel, and the constant defining the remainder depends only on  $r$ . This made it possible to obtain asymptotically sharp estimates for the error of the Gauss quadrature formulas on the classes  $W_1^r$ . In 2005 V.P. Motornyi and O.V. Motorna obtained an asymptotically sharp estimate of the best one-sided approximation by algebraic polynomials in the mean of truncated power functions, proving that, for any positive integer  $r$  and  $a \in (-1, 1)$ , there exist algebraic polynomials  $P_{n,r,a}^\pm(x)$ ,  $P_{n,r,a}^-(x) \leq f(x) \leq P_{n,r,a}^+(x)$ , such that

$$\left\| \frac{(x-a)_+^{r-1}}{(r-1)!} - P_{n,r,a}^\pm(x) \right\|_1 = \frac{(\sqrt{1-a^2})^r}{n^r} \cdot \sup_t (\pm B_r(t)) + O\left(\frac{\ln(n+1)(\sqrt{1-a^2})^{(r-2)_+}}{n^{r+1}}\right)$$

(the constant defining the remainder depends only on  $r$ ). With the help of this estimate, V.P. Motornyi and O.V. Motorna obtained comparison theorems of Kolmogorov-Hörmander type for some asymmetric classes of functions.

In 2011, V.P. Motornyi and A.M. Pasko obtained an analogue of estimate (7) for asymmetric approximations in the mean with the weight function  $\rho(x)$  satisfying on  $[-1, 1]$  inequalities  $(\sqrt{1-x^2})^\sigma \leq \rho(x) \leq 1/\sqrt{1-x^2}$ ,  $\sigma \geq 1$ ,

$$E_n^{\alpha,\beta}(W_{1;\gamma,\delta}^r)_{1,\rho} = \|\phi_{n,r;\alpha,\beta}\|_{\infty;\gamma^{-1},\delta^{-1}} + O\left(\frac{1}{n^{r+1}}\right),$$

where the spline  $\phi_{n,r;\alpha,\beta}$  is defined as the  $r$ -th periodic integral having zero mean of  $2\pi/n$ -periodic even function  $\phi_{n,0;\alpha,\beta}$  defined on  $[0, \pi/n]$  with the help of equality

$$\phi_{n,0;\alpha,\beta}(t) = \begin{cases} \alpha, & 0 \leq t \leq \frac{\beta\pi}{(\alpha+\beta)n}, \\ -\beta, & \frac{\beta\pi}{(\alpha+\beta)n} < t \leq \frac{\pi}{n}, \end{cases}$$

and the constant, defining the remainder, depends only on  $r, \alpha, \beta, \gamma, \delta$  and  $\sigma$ .

At the age of 80, Vitalii Pavlovych is in the prime of his creative powers and continues to work actively in the field of Approximation Theory and mathematical education. The staff of the Department of Mathematical Analysis and Functions Theory wishes Vitalii Pavlovych strong health and new creative successes.

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