In memoriam

In memoriam: Herbert Stahl
August 3, 1942–April 22, 2013

Fig. 1. Herbert Stahl in June of 1989 in Columbus, Ohio, at PaulN's house.

After a prolonged and arduous fight with cancer, our beloved Herbert Robert Stahl[1] died in his 71st year on April 22, 2013, in Berlin.

Herbert was born on August 3, 1942, in Fehl-Ritzhausen, in the German state of Rheinland-Pfalz. At the age of 16, he started to work as an electrician for Allgemeine Elektricitäts-Gesellschaft (AEG, General Electricity Company), and by the time of his retirement in 2008 he established himself as one of the most prominent, respected, and honored mathematicians in approximation theory, orthogonal polynomials, and related fields.

Before his university education, after working at AEG in 1958–1964, he became a sailor for a short time. His undergraduate, graduate, and postgraduate studies were all done at the Technische Universität Berlin (TUB) in the period 1965–1974. His Ph.D. (Dr. rer. nat.) dissertation in 1974, written under the supervision of Christian Pommerenke, was titled "*Contributions to the convergence problem of Padé approximants*". Eventually he became an expert on Padé approximation, but his interest was wider, it included function theory, complex approximation, rational approximation, approximation with varying weights, and orthogonal polynomials. He obtained

---

[1] Herbert never used his middle name Robert in his professional life although some of his papers appeared with his middle initial.

his second Ph.D. (Dr.-Ing.) in 1981 from TUB's Department of Information Technology. Herbert habilitated in 1984, and received his Venia Legendi,[2] apparently simultaneously, both in *Statistics and EDV*[3] and in *Mathematics* at TUB. He held various positions at TUB, including a Full Professorship in the Department of Information Technology,[4] but spent most of his career from 1986 until his retirement in 2008 at the Technische Fachhochschule Berlin.[5]

Herbert published about 90 papers and two books. One of the books and a small fraction of his papers were in statistics, the rest in mathematical analysis. He systematically applied potential theoretical and function theoretical arguments in various far lying areas; he was regarded as an absolute authority on potential theory, complex analysis, and Riemann surfaces.

He had an extraordinarily friendly personality with special fondness for music and poetry, and strong interest in politics.[6]

Herbert was an avid runner with several completed Marathons; he took running very seriously as attested by the following (rather low resolution) picture that he posted on both his professional and personal websites.

Fig. 2. Herbert Stahl the runner.

Herbert's name will long be remembered for solving several outstanding conjectures; many of his seminal works opened new channels for future research. In what follows, we discuss some of his most remarkable achievements.

Nuttall's conjectures. Let f be a function holomorphic at infinity, say,

$$f(z) = \sum_{k=0}^{\infty} \frac{f_k}{z^k}. \tag{1}$$

A *diagonal Padé approximant* to f is a rational function $[n/n]_f = p_n/q_n$ of type (n, n) that has maximal order of contact with f at infinity.

Assume now that the germ (1) is analytically continuable along any path in $\mathbb{C} \setminus A$ for some fixed set A. Suppose further that this continuation is multi-valued in $\overline{\mathbb{C}} \setminus A$, i.e., f has branch-type singularities at some points in A. This class of functions is denoted by $\mathcal{A}(\overline{\mathbb{C}} \setminus A)$.

---

[2] This term refers to an authorization to teach at a university.

[3] EDV must be Elektronische Datenverarbeitung, that is, data processing. In those days EDV basically meant computer science in general.

[4] He even co-advised a mathematically non-googlable student there, Bernhard Spiegel, who graduated in 1984 with a Dr.-Ing. degree; search www.genealogy.ams.org.

[5] In 2009, TFH-Berlin was renamed Beuth Hochschule für Technik Berlin.

[6] It appears that the latter may have been the reason why the rigid German academic establishment prevented him to occupy a position that, from the scientific point of view, he would have richly deserved.

In 1976 Nuttall has conjectured that *if A is finite then for every function $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A)$ with a finite number of branch points and with an arbitrary type of branching singularities at those points, the diagonal Padé approximants converge to f in logarithmic capacity, that is, for every $\varepsilon > 0$,*

$$\lim_{n \to \infty} \operatorname{cap}\left(\{z \in K : |f(z) - [n/n]_f(z)| > \varepsilon\}\right) = 0, \quad K \subset \overline{\mathbb{C}} \setminus \Delta, \tag{2}$$

away from the system of cuts Δ characterized by the property of minimal logarithmic capacity, i.e.,

$$\operatorname{cap}(\Delta) = \min_{\partial D : D \in \mathcal{D}_f} \operatorname{cap}(\partial D), \tag{3}$$

where $\mathcal{D}_f$ denotes the collection of all connected domains containing the point at infinity in which f is holomorphic and single-valued; see [3, REF].[7]

Thus, in his conjecture, Nuttall has put forward the important relation between the *maximal domain* where the multi-valued function f has single-valued branch and the *domain of convergence* of the diagonal Padé approximants for f constructed solely based on the series representation (1). The Padé approximants, which are rational functions and thus single-valued, approximate a single-valued holomorphic branch of f in the domain of their convergence. At the same time most of their poles tend to the boundary of the domain of convergence and the support of their limiting distribution models the system of cuts that makes the function f single-valued.

The complete proof of Nuttall's conjecture, even in a more general setting, was taken up by Stahl. In a series of fundamental papers [15,16,36,66][8] for a multi-valued function $f \in \mathcal{A}(\overline{\mathbb{C}} \setminus A)$ with $\operatorname{cap}(A) = 0$ (no more restrictions!) he proved the following.

 (i) Proved the *existence* of a domain $D^* \in \mathcal{D}_f$ such that $\Delta = \partial D^*$ satisfies (3).
(ii) *Proved weak, that is, n-th root asymptotics* for the denominators (q_n) of the Padé approximants $[n/n]_f$ of the form

$$\lim_{n \to \infty} \frac{1}{n} \log |q_n(z)| = \int \log |z - t| \, d\omega_\Delta(t) =: -V^{\omega_\Delta}(z), \qquad z \in D^*,$$

where V^{ω_Δ} is the logarithmic potential of the equilibrium measure ω_Δ, that is, $V^{\omega_\Delta} = $ const. a.e. on Δ.
(iii) *Proved the convergence theorem* (2) conjectured by Nuttall.

In his approach Stahl introduced the following local characterization of the compact Δ of minimal capacity:

$$\frac{\partial V^\lambda}{\partial n_+} = \frac{\partial V^\lambda}{\partial n_-}, \qquad \text{a.e. on } \Delta,$$

where $\frac{\partial}{\partial n_\pm}$ denote the normal derivatives on Δ. After Stahl's name, this property of compacts now is called the *S-symmetry property* and it plays a prominent role in modern constructive approximations of analytic functions in the complex plane.

General orthogonal polynomials. In the more than two hundred years of the theory of orthogonal polynomials the first hundred years were devoted to special, so called, classical systems. In the first half of the 20th century more general weight functions emerged, but still the support of

---

[7] The letters "REF" refer to the short list of references that precedes Herbert's publication list.
[8] These refer to Herbert's publication list where the letters "HS" are prepended to them.

those weight functions was mostly the interval $[-1, 1]$ or the unit circle. Then, through the works of P. Erdős, H. Widom and J. Ullman, the theory of orthogonal polynomials with general support started to evolve. The book [45] by Stahl and Totik laid the foundation of the theory of "General Orthogonal Polynomials", namely, when the support of the generating measure ρ is an arbitrary compact subset of the plane. The book uses potential theory to establish some basic properties that all orthonormal systems share. For example, if $p_n(z) = \gamma_n z^n + \cdots$, $\gamma_n > 0, n = 0, 1, \ldots$, are the orthonormal polynomials with respect to ρ and if S is the support of ρ, then for the leading coefficients γ_n it is always true that

$$\liminf_{n \to \infty} \gamma_n^{1/n} \geq \frac{1}{\operatorname{cap}(S)}, \tag{4}$$

where $\operatorname{cap}(S)$ denotes the logarithmic capacity of S, and

$$\liminf_{n \to \infty} |p_n(z)|^{1/n} \geq e^{g_{\overline{\mathbb{C}} \setminus S}(z)}, \qquad z \notin \operatorname{Con}(S),$$

where $g_{\overline{\mathbb{C}} \setminus S}(z)$ denotes the Green's function of the unbounded component of $\overline{\mathbb{C}} \setminus S$ with pole at infinity and $\operatorname{Con}(S)$ is the convex hull of the set S. It turns out that there is a large family of measures for which the orthonormal polynomials "behave decently" in the sense that their behavior resembles that of the classical orthogonal polynomials. This family, called **Reg**, is defined by the minimality property

$$\lim_{n \to \infty} \gamma_n^{1/n} = \frac{1}{\operatorname{cap}(S)}, \tag{5}$$

and under mild conditions on ρ this property is actually equivalent to either of

(i) the asymptotic zero distribution of the orthogonal polynomials (p_n) is the equilibrium distribution of the support S,

(ii)
$$|p_n(z)|^{1/n} \to e^{g_{\overline{\mathbb{C}} \setminus S}(z)}, \qquad z \notin \operatorname{Con}(S),$$

(iii) for all $0 < q < \infty$ or at least for one such q,

$$\sup_{P_n} \frac{\|P_n\|_S^{1/n}}{\|P_n\|_{L^q(\rho)}^{1/n}} \to 1.$$

A major emphasis of [45] is on the **Reg** class and on various regularity criteria, i.e., criteria for $\rho \in$ **Reg**. It also contains several applications of regularity concerning rational interpolation and approximation, approximation theory, and orthogonal polynomials with varying weights. Since then regularity in this sense has been used in different situations; it gives a weak global condition under which many properties of orthogonal polynomials can be localized. Stahl-Totik's [45] itself has become a standard reference tool.

Rational approximation of $|x|^\alpha$. About one hundred years ago S. N. Bernstein proved that if

$$E_n = \inf_{P_n} \| |x| - P_n(x) \|_{[-1,1]}$$

is the rate of the best uniform approximation of $|x|$ on $[-1, 1]$ by polynomials P_n of degree at most n, then the limit

$$\lim_{n \to \infty} n E_n = \sigma > 0$$

exists. In the 1930's and 1940's he extended this to approximation of $|x|^\alpha$ when $\alpha > 0$ is not an even integer, in which case the corresponding limit was a number $\sigma_\alpha > 0$ that depends on α. The exact value of σ and σ_α is still not known, so the following very precise result concerning the corresponding quantities for rational approximation came as quite a surprise.

Let ρ_n be the best uniform approximation of $|x|^\alpha$ on $[-1, 1]$ by rational functions $R_n = P_n/Q_n$, where P_n and Q_n are polynomials of degree at most n, that is,

$$\rho_n = \inf_{R_n} \left\| |x|^\alpha - R_n(x) \right\|_{[-1,1]}.$$

For these Stahl proved in his Acta Mathematica paper [76] the beautiful relation

$$\lim_{n \to \infty} e^{2\pi \sqrt{\alpha n}} \rho_n = 4^{1+\alpha/2} \left| \sin\left(\frac{\pi \alpha}{2} \right) \right|. \tag{6}$$

Prior to this, there have been numerous partial results leading to (6). The whole subject started with D. J. Newman who, in 1964, noticed that rational functions of a given degree can approximate $|x|$ much faster than polynomials. Namely, for the case $\alpha = 1$ he proved the inequalities

$$\frac{1}{2} e^{-9\sqrt{n}} \le \rho_n \le 3 e^{-\sqrt{n}}, \tag{7}$$

see [2, REF]. This result started a new chapter of approximation theory, and subsequently the estimate in (7) has been improved in various papers. Eventually T. Ganelius and N. S. Vyacheslavov showed that for rational α the quantity $e^{2\pi \sqrt{\alpha n}} \rho_n$ remains in between two positive constants. That the limit in (6) exists and equals the right-hand side was conjectured by R. S. Carpenter and R. S. Varga using high precision calculations. The full verification of (6) for all α put an end to a long line of research and is certainly one of the top results in approximation theory.

The BMV conjecture. Let A and B be $n \times n$ Hermitian matrices, with B being positive definite. If $\mathrm{Tr}(H)$ denotes the trace of a matrix H, then the Bessis-Moussa-Villani conjecture from 1975 claims that the function $h(x) = \mathrm{Tr}(\exp(A - xB))$ is completely monotone in x, i.e., it is the Laplace-transform of a positive measure σ on $[0, \infty)$, see [1, REF]. In other words,

$$h(x) = \int_0^\infty e^{-xt} d\sigma(t). \tag{8}$$

The importance of the conjecture lies in the fact that it implies $(-1)^m h^{(m)}(x) \ge 0$ for all nonnegative real x and for all nonnegative integer m,[9] and, hence, the conjecture generates a series of trace inequalities for Hermitian matrices.

This so called BMV conjecture has attracted much attention in the mathematical physics community due to the fact that it implies several inequalities for the so called quantum partition function. Several reformulations have been known. For example, it is equivalent to the fact that the polynomials $\mathrm{Tr}((A + xB)^m)$ have nonnegative coefficients for all natural m. The conjecture has been verified in various special cases. It had even been known that (8) is true with some explicitly given signed measure σ, but the positivity of σ is an essential part of the conjecture and this had eluded all efforts until 2011, when Stahl proved the conjecture in full generality. Stahl put the solution to arXiv in the middle of 2011, and a vastly simplified version a year later. He submitted the solution to Acta Mathematica, and received a positive response only a few months before his death, see [89].

---

[9] Actually, by Bernstein's theorem, the latter property is equivalent to complete monotonicity.

Reminiscences by friends

Alexander Aptekarev. I met Herbert Stahl for the first time in Poland at a conference that was organized by Jacek Gilewicz. It was during the nice sunny Summer of 1985. Herbert's series of papers in which he proved his first outstanding result on convergence in capacity of the diagonal Padé approximants for multivalued function had not appeared yet, but were to be published that year. However, we had in Moscow preprints of his result and I studied his proofs with the help of Zhenya Rakhmanov; perhaps Zhenya was involved in the refereeing process of Herbert's papers. My first words to Herbert were about details of his proof and he was pleasantly surprised about it. Next day we had a dinner together and discussed Hermite-Padé approximants. Herbert just came from Canada, where he had visited John Nuttall who had just published in JAT his remarkable paper on this topic. I asked Herbert about the distribution of poles of Hermite-Padé approximants for two Markov-type functions with overlapping supports. His answer was "no problem, I know it". He took a napkin and a pencil and fell down in thinking nirvana. After 5–10 minutes he woke up and said "yes, I know it, but there isn't enough space on this napkin to write it down". Then several years later he wrote a sketch of results regarding that problem. It was a fantastic manuscript about a hundred pages long, divided into two vertical columns, performed with calligraphic handwriting and with fine drawings of figures (plenty of pictures, like ancient engravings). It is a pity that his results were never published.

Fig. 3. Herbert Stahl & AlexanderA in July of 2012 in Leuven, Belgium.

From that that moment on, Herbert has became my very best friend, colleague, and teacher in various matters. I cannot stop reminiscences: "visits eastern Berlin's small theaters through the destroyed Berlin's wall", "Herbert is skating in Moscow's Gorky Park", "discussions after the International Congress of Mathematicians in Madrid", ..., "the last time in April of 2013, Hector Str. 12, Berlin-Halensee".

Laurent Baratchart. I first met Herbert Stahl in 1990, at the US-USSR Conference on Approximation Theory organized by E.B. Saff and the late A.A. Gonchar in Tampa, Florida. Already

then, Stahl's theorem on extremal domains of convergence in capacity for Padé approximants to functions with branchpoints had gained, in function theoretic circles, the high reputation it deserves. As stressed by A.A. Gonchar in his address, it was one of the most penetrating results in rational approximation, in some sense completing the analysis of Padé approximants to functions with polar singular sets initiated by J. Nuttall some ten years earlier. I was hearing about the subject for the first time and the core of Herbert's analysis, yielding weak asymptotics of non-Hermitian orthogonal polynomials on symmetric contours, was far beyond my understanding. However, I could feel he was a prominent figure in the field and I remember noticing that, in the eyes of many colleagues, Herbert was a character tinted with mystery. This impression never cleared up completely.

I came to know him better three years later, at a conference in Antwerp. At that time his landmark book with V. Totik on orthogonal polynomials was receiving rapid recognition and, like most of my colleagues, I was impressed by the depth of Herbert's achievements. A pleasant gentleman with a tall, classy figure, he appeared to unfold some of the most important issues in the field without ever loosing his elegant style and good humour. He was quite eloquent as we spoke about Belgium history, and I was struck by his vast culture and his long distance driving.

I started to discuss Mathematics in earnest with Herbert a year later, at CMFT 1994 in Penang. This is when I came to realize his rather broad spectrum of interests. A student of C. Pommerenke, he was foremost a complex analyst with superior geometric insight, and one of the finest experts in potential theory I had met. But he was also curious of applications, attracted by numerical aspects to the point where he would write programs, and be knowledgeable in time series analysis, modeling, differential equations, which he had obviously practiced and taught before. Maybe this background helped him with the patience necessary to listen to my explanations, and with F. Wielonsky we started collaborating on uniqueness issues and strong asymptotics in best L^2 rational approximation to Markov functions. We typically had long, informal conversations, full of non-Mathematical digressions, and he would then undergo periods of solitary thinking after which he came out with a new idea.

In 1999 Herbert spent a sabbatical leave in Sophia Antipolis, with F. Wielonsky and myself. He was amused by the fact that a street in Antibes was named "rue de l'abbé Stahl." This is when I started to really understand, and perhaps fully appreciate his approach to minimal capacity contours. I was fascinated by his beautiful analysis of limiting n-th root errors and by his geometric interpretation of them as equilibrium potentials. Beyond Mathematics, Herbert's stay on the Côte d'Azur was the occasion of long conversations on history, politics, Europe, German mood, psychology, Italian art, and French cuisine. Everyone in our department remembers a cheerful and cultured guest, particularly good at telling jokes to support his views and never at a loss for a smart answer. He seldom spoke about himself and only hinted at personal experiences, but they must have been manifold.

His stay with us laid ground for further common investigation of best rational and meromorphic approximation to algebraic functions. The bottom line was to exploit weighted non-Hermitian orthogonality which unexpectedly arises in this context, even though interpolation is no longer an initial concern. I think Herbert liked this resurgence of a subject to which he had contributed so much. I went to Berlin several times afterwards to visit him, and from our long strolls through the city I recall his mischievous look at life and witty comments on Mathematics and society, both amusing and grave. I can still see him walking down Tiergarten's alley talking about German affairs and rhododendron blossoms before heading straight to winding numbers of error curves. He was always concerned to put things in historical perspective, and I learnt a lot from these encounters, both Mathematically and otherwise. Later, M. Yattselev joined our

collaboration and we had some exciting times which left Maxim and myself with a set of notes in Herbert's tight writing, still to be analyzed.

As far as I know, Herbert has had various Mathematical interests in the 2000's, most of them connected with extremal potential problems and orthogonal polynomials. He also refurbished older material, which he felt had not reached full aesthetic balance. He loved to write and was quite demanding, on himself and co-authors, regarding exposition. From 2008 on, he got strongly interested in the Bessis-Moussa-Villani conjecture on exponentials of Hermitian matrix pencils, "for sport" he said though there may have been other reasons. His views on this problem evolved much over the next two years, judging from rare occasions where he would bring up the issue, and he was able to prove the conjecture in 2011 but would not see the paper in print. In this last Mathematical deed, he answers with a lucid piece of complex analysis a question originating from statistical quantum mechanics which resisted attempts for more than 30 years.

Although Herbert was not striving for honours, I believe he was moved on the occasion of a conference organized by V. Totik in Szeged, June 2012, where many talks paid tribute to his work. This was less than a year before his untimely passing away on April 22, 2013. From my last visit to see him, two weeks before he died, I remember his courage.

We lost a friend, an understanding colleague, and a penetrating Mathematician.

Bernhard Beckermann. I was very sad to learn about Herbert Stahl's passing since he influenced quite a bit my way of doing mathematics. It was probably around the end of my Ph.D. thesis in 1989 with Günter Mühlbach at the University of Hannover that I had the first opportunity to see Herbert at a colloquium talk. It was always very exciting for me to listen to his talks that were remarkably clear, giving concepts and ideas, without too many technicalities.

It was Herbert and Ed Saff who introduced me to the wonderful world of logarithmic potential theory. This took place at an Oberwolfach meeting, a year after I joined in 1993 the group of Claude Brezinski in Lille. I gave a talk about lower bounds for the condition number of positive definite Hankel matrices where particular orthogonal polynomials with varying weights were used to prove sharpness. Herbert has been extremely encouraging to me, curious about my results, and showed me how my (rather elementary) computations fitted in a nice general theory. I should explain that until then my scientific work was rather restricted to algorithmic and numeric aspects of approximation theory, though during my studies I took quite a few complex analysis classes with Professors Mues and Tietz. Herbert allowed me to discover how things are nicely related, and, since then, I really like to study approximation in the complex plane.

Subsequently, Herbert visited me in Lille, that was our first attempt to write a joint paper; several others followed, unfortunately without finalizing. When he visited my family, we chatted also about a job offer in a bank in Germany which I just got from a former economy student of mine – I have to admit that I did not follow Herbert's advice. Lille (and Berlin) are quite close to Belgium, where many activities took place on orthogonal polynomials, but also several OPSFA's[10] and the series of conferences on OP organized by the GSF group of Lasser in Munich gave opportunities for exchanges. I remember especially well his participation in the second and third edition of our Journées Approximation here in Lille where he gave a talk, and together with Alexander Aptekarev and Valery Kaliaguine we had quite deep discussions afterwards about the numerical computation of rational approximants and, in particular, Padé approximants. Unfortunately, his expertise in this domain never resulted in a paper except for some general remarks on

---

[10] Orthogonal Polynomials, Special Functions and Applications

"spurious pole clearing" procedures. I should also mention the very fruitful collaboration in the framework of two INTAS networks (2001–2007). There are far too many joint memories to tell them all. Maybe just the one where I was impressed how easy it was for Herbert to organize in 24 hours 20 tickets for the Bolshoi theatre, where we spent a wonderful evening during the Moscow INTAS meeting. And how friendly the atmosphere was in the 2012 Szeged meeting organized by Vilmos Totik in honor of Herbert's 70th birthday, with a lot of singing in the evenings.

I spent quite some time to think about Herbert's many deep and inspiring mathematical results. Just to number one of them, his striking example of spurious Padé poles everywhere in the plane for a simple modification of a Markov function on $[-1, 1]$. Herbert was not only an extraordinary mathematician, but also had an extraordinary personality, always friendly and charming, very open-minded, somebody who loves to share ideas, be it about mathematics, politics or any other aspect of life. It is extremely important for a mathematical community to have people like Herbert Stahl. Both I and my colleagues here in Lille will miss him.

Hans-Peter Blatt. The first time I met Herbert Stahl was at the conference on Constructive Approximation of Functions in Blagoevgrad, Bulgaria, in 1977. During this conference I realized that Herbert Stahl had excellent contacts to the mathematicians at the Steklov Institute in Moscow around A.A. Gončar working in the field of rational approximation, especially Padé approximation. His wide connections to Russian, American, and European mathematicians were a great help and support for quite a number of colleagues to obtain new results in rational approximation, combining classical approaches with methods of potential theory and complex analysis. Herbert Stahl's deep results on the characteristics of best approximants have stimulated many researchers to obtain new techniques and results in rational approximation. For example, the close connections between poles and zeros of best approximants and their region of convergence could be reflected in a new light.

I remember very well our joint stay during the semester of Complex Analysis at the Polish Academy of Sciences in Warsaw in 1992, an opportunity for both of us to combine mathematics and musical interests by visits of the Opera House in Warsaw. At the Conference in Acquafredda di Maratea in the same year, I encouraged Herbert to visit Rome after the conference by pointing out to him the comfortable but affordable accommodation in the Casa Kolbe near the Capitol.

Herbert was a warm and open friend and an excellent mathematician. His results in the constructive theory of functions will keep their deep impact in the future.

Diego Dominici. It is with a heavy heart that I take up my pen to write these last words in which I shall ever record the singular gifts by which my friend Herbert Stahl was distinguished. I met Herbert for the first time at the OPSFA9 conference in Luminy in 2007. It was the first major conference that I attended, and as such, I was (and still remain) an obscure figure, unknown to most of the people attending. And yet, upon hearing that I would be in Berlin, Herbert gave me a card with his personal phone number and insisted that I call him as soon as I would arrive. I did so, and he spent a whole afternoon showing me around Berlin. He made me feel so special, as if I would be the most important visitor he ever had. This was not an accident, as I discovered in my subsequent encounters, but just a measure of his immense generosity. He would openly offer his help and advice from everyday things to mathematical questions.

Of his work and legacy as a mathematician I will let others, with more expertise than I, talk about it. I will just mention one particular problem he solved and that I had the privilege of witnessing from beginning to end. Our group in Berlin, directed by Olga Holtz, was very interested in finding ideas to prove the BMV conjecture [1, REF]: for every $n \times n$ Hermitian matrix A and B,

where B is positive semidefinite, there exists a positive measure μ such that

$$\operatorname{Tr} \exp\left(A - \lambda B\right) = \int\limits_0^\infty e^{-\lambda t} d\mu\left(t\right), \qquad t > 0\,.$$

Herbert heard about this problem in one of our seminars and immediately suggested a possible way of attack. In subsequent meetings (over coffee and beer), he told me of his progress and sometimes of his frustration, but it was obvious that his path was always very clear to him. I was amazed by the power of his analytical mind and the breadth of his knowledge. When his work was finished, he hesitated for quite a while before writing his results. I was very happy for him when his preprint finally appeared on the arXiv, see [89].

I can only conclude by saying that with his passing the world has lost a wonderful human being, and I have lost a friend whom I shall ever regard as the best and the wisest man whom I have ever known.

Kathy Driver. I met Herbert at an Approximation Theory meeting in Kecskemét, Hungary, in August of 1990 when I was a Ph.D. student with advisor Doron Lubinsky.

Fig. 4. Herbert Stahl & Kathy Driver 18 years later in September of 2008 in Leganés, Spain.

Claudia Cottin and Margareta Heilmann were friendly to a stranger from South Africa and invited me to join them and Herbert for dinner on the evening before the conference started. At the end of the first day of the conference, Herbert asked me if I was interested in Hermite-Padé approximation and, without waiting for me to answer, he spent the next few hours with a pencil and paper (I still have the sheets of formulas) introducing me to Type I and Type II H-PA's. He had a curious "pencil-and-paper" technique which consisted of dividing the page into vertical columns, very frugal and using up every inch of the page! In early 1991, he visited Johannesburg to work with Doron and gave me a copy of Nikishin's paper to read. We proved an interesting but incomplete result on the normality of multi-indices for Nikishin systems of n functions and started a longer project on the properties of simultaneous Hermite-Padé approximants to Nikishin systems. Herbert had so many good ideas about simultaneous H-PA's, he seemed to know everything already and it was difficult to keep up with him! The normality (or not) of all multi-indices for a Nikishin system in the special case $n = 2$ still intrigued me and I was happy to find a proof Herbert liked very much that a Nikishin system of two functions is perfect. We

co-authored 4 papers on Nikishin systems and wrote another paper some years later in response to a question after my talk in Irsee in 2004 relating to the measure of orthogonality when an orthogonal sequence is generated by n and $n - 1$ given, fixed interlacing points on the real line, using Wendroff's Theorem. After normalization, all the polynomials in the sequence of degree $\leq (n - 2)$ are uniquely determined but the polynomials of degree $\geq (n + 1)$ are generated by a three-term recurrence relation. Herbert had a good idea and I did all the work!

It was a great privilege to be Herbert's friend and share his company. Apart from his wonderful mathematical ideas, he was also a source of (sometimes ancient) human historical facts: every statue in every town has its rationale, every painting tells a story beyond its artistry, every community of people has its unique modus vivendi. He was a brilliant light in our community and his many friends will miss him terribly.

Antonio J. Durán. Each time I met Herbert from September of 1997 on, he reminded me of his gratitude for inviting him to deliver the opening lecture in a meeting on orthogonal polynomials and special functions that I had organized in Seville that year. The opening lecture took place at the impressive Old Tobacco Factory of Seville. Constructed at the beginning of the XVIII century, that tobacco factory was the first one in Europe but nowadays it is the central building of the university. The reason for Herbert's appreciation was neither related to tobacco nor to the university, but to opera. It turns out that the old tobacco factory of Seville was the place where Carmen, the heroine of Bizet's opera, worked. Herbert loved opera, and as he reminded me many times, he felt a profound sentiment while lecturing on Padé approximants in the heart of Carmen's workplace. Herbert, besides being a great and influential mathematician, was also a warm and grateful friend. A few years later, he returned me the favor. It was in Moscow where we had met at one of those INTAS meetings. He knew the city very well, and, in a certain sense, he officiated as my cicerone part of the time during this visit. One evening, he took me to the Bolshoi where we, together with some other colleagues, enjoyed Verdi's opera, Nabucco. " 'Immortality' may be a silly word", wrote Hardy in his *A mathematician's apology*, "but probably a mathematician has the best chance of whatever it may mean". Herbert was a mathematician, a true one, and he will live in the depth of his mathematics forever.

Guillermo (Bill) López Lagomasino. Brilliant, elegant, generous, friendly, polite, modest, all that and more was our dear friend Herbert. We met during the OPSFA conference held in Segovia, Spain, in 1986.[11]

Of course, I already knew some of his early contributions to orthogonal polynomials in the complex plane, mainly through beautifully handwritten manuscripts that had reached A. A. Gonchar and I was eager to meet him. My expectations were surpassed for he was not only a gifted mathematician but an exceptional human being. We made friends rapidly (who didn't). He was a great talker and a good listener and we spent many hours during lunch and after sessions discussing mathematics and politics. In 1988 on my way to Moscow from Bremen he invited me and my daughter to spend several days in his house in Berlin. My daughter, still a teenager, was also impressed but for other reasons. He showed us around the city and during the very few hours we had for ourselves Herbert insisted in lending me his car to drive around (which I never dared to do). In each conference we went to it was a treat to spend time chatting with him and hear his views on whatever event was on the spotlight. About 5 years ago, I had the opportunity to invite him for a month to Carlos III University in Leganés (Madrid) to give some lectures

---

[11] PaulN adds: Bill and Herbert already met in Bar-le-Duc, France, in October of 1984.

Fig. 5. Herbert Stahl in September of 1986 in Segovia, Spain.

for graduate students on potential theory in my course "Orthogonal Polynomials and Rational Approximation". I attended those lectures and enjoyed his presentations on the subject for their motivation, precision, and accuracy. By then he was already ill and mentioned he had some blood problems, which he was taking care of with a special diet. "Nothing to worry about", he said. We invited him for dinner twice and each time showed up with a big bouquet of flowers for my wife. She was also impressed with his kind manners and politeness. We are missing you Herbert.

Doron S. Lubinsky. Do you remember Herbert Stahl's elegant handwriting? If you attended his conference talks, you could not help but be impressed by his slides, with their beautifully written cursive. That attention to detail was also evident in his papers, and in his warm considerate personality. He was thoughtful and polite, as well as friendly - quite apart from his great mathematics.

No doubt, other obituaries will focus on his breakthroughs on rational approximation, on extremal domains, and orthogonal polynomials, as well as his definitive monograph with Vili Totik. My own collaboration with Herbert was restricted to two topics: interpolatory integration rules and biorthogonal polynomials. The former was stimulated by a question of Tom Bloom: what distributions are possible for the abscissa of a convergent sequence of interpolatory integration rules on $[-1, 1]$? It turned out that half the points have to have arcsine distribution, and the rest may be somewhat arbitrary. Moreover, as one increases the precision of the integration rules, the proportion of zeros having arcsine distribution increases - until the limiting case of Gauss quadrature, where all absicssa have arcsine distribution. Various aspects of this question were considered in a series of joint papers with Herbert, Tom Bloom, and myself. Herbert's potential theoretic expertise provided the key ideas.

Our second collaborative theme was stimulated by Herbert's refereeing of a joint paper of Avram Sidi and myself, on polynomials that are orthogonal to powers of $\log x$. In discussions

at a workshop in Hungary honoring a birthday of Erdős, Herbert raised a conjecture about the zero distribution in the general case, where $\log x$ is replaced by a general increasing function. We started collaboration on a paper in 1995, and the paper went through various drafts, but was never finished. A version of this will be submitted to the memorial issue of JAT. We did complete one paper on explicit representations of polynomials that are orthogonal to powers of x^α for $\alpha > 0$.

Herbert was invited to deliver a plenary talk at the conference "Constructive Functions 2014", in Nashville, but emailed his doubts that he would be able to attend. Sadly, he was correct. However, it was heartening when Pierre Moussa recently gave a talk at Georgia Tech, on the famous BMV conjecture [1, REF], and spent most of the time discussing Herbert's remarkable solution of this problem. It is fitting that this breakthrough, will appear in Acta Mathematica, joining some of his other great papers there.

Francisco (Paco) Marcellán. I was very sad when I received the bad news that our friend Herbert R. Stahl passed away in April 22, 2013. He was fighting against a pancreatic cancer during the last years and one of his more brave actions was to continue doing mathematics and attending meetings of approximation theory and orthogonal polynomials. May 2102 at Szeged was a very good opportunity to share his enthusiasm with many people of our scientific community.

Let me recall the first time I met Herbert. It was on the occasion of the Bar-le-Duc meeting held in October of 1984. I was very impressed with is lecture "On the divergence of certain Padé approximants and the behavior of the associated orthogonal polynomials". Herbert constructed a non-positive weight on $[-1, 1]$ such that the diagonal sequence of Padé approximants at infinity of its Cauchy transform has poles which are dense in the complex plane. Thus, the corresponding Padé approximants do not converge locally uniformly anywhere in the complex plane. I was always impressed by his careful presentations, particularly when he used slides with an overhead projector, containing statements of different colors written with his beautiful handwriting.

I met Herbert two years later in Segovia, Spain, in September of 1986, where we organized a meeting on orthogonal polynomials and applications. This was the first time that the Spanish team was involved in these activities on an international level. One of the organizers was José Luis Rubio de Francia, one of the leading Spanish mathematicians in the last 40 years, who, unfortunately, died of cancer in January of 1988 at the age 38. Herbert helped E. A. Rakhmanov to find the site of the meeting because of an unforeseen event that happened to Zhenya on the train between Madrid and Segovia. The organizers were very worried because they had no news from Zhenya, not even whether he actually arrived in Spain. In the last moment, we changed the schedule of the plenary speakers because we anticipated that he cannot attend the meeting. We were very happy when Herbert carried Zhenya to the lunch after the opening ceremony. Zhenya invited Herbert, Bill López Lagomasino, and myself to taste a good salami as well as some Russian drinks in his room once we have concluded the sessions of that first day. That was the first time I had the opportunity to learn a lot from Herbert about politics, literature and travel, as well as to observe his keen knowledge of the Soviet Union.

Herbert was very active in our community. He attended all the OPSFA meetings and we enjoyed nice moments with him.

His book "General Orthogonal Polynomials", coauthored with Vilmos Totik, was a milestone in the development of the theory and had a very strong influence in the asymptotics of polynomials orthogonal with respect to new families of measures.

The last and, in my opinion, most important contribution of Herbert was the proof of the BMV conjecture stated in 1975, see [1, REF]. When I read it in ArXiv (August 17, 2012) I was much impressed by the solution that beautifully displayed Herbert's approach to mathematics. His

creativity and deep analysis of the problems coupled with careful presentations were character-istics of his mathematical talent. I would like to conclude with some words of a Spanish singer "*did not know how to react of seeing that a piece of us was leaving without feeling that our heart died too*".

Farewell our comrade and beloved friend Herbert.

Andrei Martínez Finkelshtein. Until 1994 Herbert Stahl was just a series of impressive manuscripts I borrowed from Guillermo López and Andrei Gonchar, and which I had to suffer through during my graduate school. They were impressive both by their content and their form. They were actually MANUSCRIPTS of the pre-TeX and even pre-ChiWriter era: handwritten text with carefully drawn illustrations. But they looked almost like typewritten with the "hand-written" font: a perfectly aligned text, not a single correction, with not a word crossed out or visibly erased. I remember that I thought that the guy who wrote them should have a remarkable mental clarity.

The content of those manuscripts was no less impressive, giving the characterization of the extremal compact sets on the complex plane in terms of the S-property ("S" naturally from "symmetry", but for me "S" fairly standing for "Stahl") and establishing the zero asymptotics for polynomials satisfying non-hermitian orthogonality conditions (in consequence, proving the convergence of Padé approximants to functions with branch points). The text was both beautiful and frightening, at least for a Ph. D. student I was then. The main ideas were simple and elegant (thus, beautiful), but technical difficulties clung to them like shells to the bottom of an old boat. I remember Jenya Rakhmanov encouraging me: "if you are able to understand these papers together with my last paper with Gonchar (Equilibrium Distributions and Degree of Rational Approximation of Analytic Functions), then you are 90% through your degree". Now I understand that Herbert had a built-in mathematical GPS that allowed him to navigate (or better to say, to fly over) the technical forest (or desert?), crossing the dunes ("lemmas") without losing the general view of the landscape.

I finally met Herbert (and Kathy Driver, together with Vasiliy Prokhorov, whom I met before in Moscow) in Zurich, in 1994, during the ICM. We immediately became friends (who didn't?), and my respect for Herbert as a mathematician was enforced by my admiration for Herbert as a human being. He was a GENTLEMAN, in the only true sense: kind, generous, elegant, discrete and polite. It was a pleasure talking to him and you never felt (or at least he never made me feel) that you were talking to somebody much smarter than you.

Stahl's mathematics had a huge impact on my own work. Not only that the whole concept of the S-property incidentally became one of the guide lines in my research, but I always found highly appealing his ability to drag any mathematical problem (either from real analysis, approx-imation theory, or linear algebra) to the complex plane, or even better, to a Riemann surface, which was his favorite playground, where the problem was ultimately solved. I saw him doing it when proving the rate of rational approximation of $|x|$ on the real line, or when proving the BMV conjecture, and actually in any single paper he wrote. Herbert felt really comfortable on a Riemann surface, surrounded by purely geometrical concepts like "vortex" or "flux".

I had a privilege to host Stahl in Almería twice, in 2008 and 2010, both times for 3 months. He probably found here a relaxing environment where he could work without pressure, enjoy his promenades along the sea and good food, and - I wish to think - our long conversations on very diverse topics. I definitely enjoyed it very much. If anything I regret is that I didn't push strong enough to have at least one of our joint project finished in a paper form, but there still might be time for this.

My family fell in love with Herbert too, especially my kids (kids, as everybody knows, have a radar for kind people). We had excursions together and spent many pleasant evenings at my place. During his last visit to Almería in January 2010 Herbert brought a traditional German Christmas gift: a vertical windmill with shepherds and angels, moved by the hot air rising from the lit candles placed beneath. Every Christmas we lit these candles, and every year my older kids used to ask me about Herbert ("the big guy with the white hair"). A year ago, when I told them that he was very ill, I was touched that they had the initiative to gather around the windmill, ask me to take a picture of them and send it to Herbert with their wishes for the prompt recovery. That is what I did. We sent him an email saying "you have always been a tough guy, and we hope that your physical and mental strength will win the disease". Well, our wish unfortunately didn't come true, and we all miss him. But as this obituary proves, his mathematics is alive and developing. His books and papers are read and re-read. And his friends remember him with love and admiration. So, long live Herbert Stahl!

Paul Nevai. As I recall, I might have been the first approximator to meet Herbert in the US. I am not sure of the year, it must have been in the early 1980s, but I remember that it was during Herbert's first visit to John Nuttall that he contacted me from London, Ontario, and told me that he wanted to come to Columbus & Tampa and asked if I was available. I heard of him before that already from Ed Saff who was very much impressed by Herbert's dissertation and I was eager to meet him. It was a little unusual that Hebert neither asked for any financial support nor wanted to give a talk. The only thing what he accepted was a university paid hotel (or perhaps motel) room. I remember we had a very long chat about the current state of orthogonal polynomials that I knew better than he because those days he was only a Padé guy. Although I don't remember it but Vili Totik told me recently that it was I who first mentioned Herbert to him when Vili came to spend 2 years at Ohio State in 1983–1985. So it is quite likely that I played a catalytic role in bringing Herbert and Vili together that ended up producing major breakthroughs in orthogonal polynomials, including their book [45].

Fig. 6. Herbert Stahl with PaulN and VilmosT in June of 1989 in Columbus, Ohio.

One curious or rather mysterious thing regarding Herbert is that we never asked him to become an associate editor of JAT even if he would have been an ideal editor. He was a great scientist, his English was impeccable, and he was well organized. I can't explain why we never approached him. In a sense he was similar to Franz Peherstorfer, except that Franz's English was not so good, and we did appoint Franz to JAT. I am just guessing, but maybe we knew that Herbert had a heavy teaching load and we didn't want to make his life more complicated than absolutely necessary.

While preparing this obituary, I found out that Herbert was an intensely private person. In a sense this was shocking news for me because I met him on numerous occasions and I never had that impression. However, in retrospect, comparing him to other approximators, I see now the difference between him, and, say, Kurt Jetter or Paco Marcellán or Giuseppe Mastroianni whose private lives and families I am very well familiar with. Knowing how private he was, I now even more appreciate that a few years before the final stages of his illness Herbert invited me to visit him in Berlin; I will regret for the rest of my life for not having done so. On the other hand, I am glad I went to Szeged in May of 2012 and had a chance to meet him before he left us for good.

Fig. 7. Herbert Stahl with SergeyK, PaulN, and AndreiMF in May of 2012 in Szeged, Hungary.

Herbert's best Marathon result what I found on the internet was 3:27:00 at the 1989 Berlin Marathon which was only 2:00 short of qualifying him for Boston and it beat both Paco Marcellán's 3:27:09 (1991, Madrid) and my own 3:27:46 (1999, Columbus) personal records. I mention this because those who are runners will share my amazement how close our personal bests are despite our very different physical attributes and our age differences when we set our records.

Let me finish by a small anecdote. Those who know me, know that one of my obsessions is studying nazism, communism, and the Holocaust. During one of my heated discussions of nazi horrors, I kept poking my fingers at Herbert's stomach. This was a subconscious act but Herbert paid attention to it and noted in a friendly and innocent way that he played no role in them. It was an awkward moment for me but I deserved it.

John Nuttall. I remember with pleasure my interactions with Herbert, which occurred prior to 1992, when I retired from my university position. We had an overlapping interest in certain aspects of the Padé approximant, but the passage of time has made it difficult for me to comment on details of this subject. However, Herbert was a joy to know, and I am happy to make a few comments on our interactions in other areas.

Herbert visited us in London, Ontario, Canada, on two occasions, one of which probably lasted a week or two. We had intensive discussions on mathematics, and I also learned something of Herbert's other interests. We shared an interest in classical music. On one of his visits he kindly gave us a gift of a plate showing a view of the old Royal Opera House in Berlin.

On his first visit my son Geoff was a first year music student at the university, and we attended a concert in which he was performing on the violin. Geoff played competently, but I got the impression that Herbert thought he was not by any means perfect, which suggested to me that Herbert was a serious student of classical music. Geoff has been lucky enough to have a

successful career, and, with his quartet, he has toured Germany several times, but unfortunately I have not heard that Herbert ever was able to see how Geoff matured as a musician.

I remember that I met Herbert, perhaps for the last time, in 1989 at the Oberwolfach Conference in the Black Forest, at a week-long conference, where we were both giving talks. What particularly stays in my mind is that Herbert showed me a preprint in which Louis de Branges claimed to describe a proof of the Riemann hypothesis. I could not understand the proof, and it was not accepted as valid by the number theory community. However, this sparked my latent interest in the subject, and five years ago I started to follow an analytic approach to the problem based on the work of Csordas, Norfolk, & Varga (1986) and Csordas & Varga (1988). I have enjoyed my efforts, which continue to this day, and I much appreciate the stimulus of Herbert that led me to this path. Recently, on the web, I came across a remarkable document by de Branges, called *Apology*, written in 2010. It would have been very interesting to hear Herbert's views on this paper.[12]

Vilmos Totik. It was in 1988 on a beautiful spring morning when I first had a discussion with Herbert in a park near Georgia Tech. We were on the same wavelength right from the beginning: he was outlining a general theory of orthogonal polynomials based on a decomposition principle. Later that principle fell through, but much of the theory could be saved, and that is when our collaboration started. I enjoyed every minute of that collaboration, and during the years I learned more from Herbert than from anybody else in my life. Rumor claimed that he was against joint papers, therefore I was not pressing on that issue, but soon it became clear that our results and efforts should be united and then the outcome would be sort of a research monograph. It was the time of fundamental changes in the Soviet block, in particular in Hungary and East Germany, and Herbert was writing me from Berlin that "it is stupid to do mathematics when people are on the Berlin wall". We finished the manuscript (in TeX) by the end of 1989, and it was the very first printout on the very first laser printer in Szeged (the quality of the print was, of course, far superior compared to previous dot matrix printers, and Béla Sz.-Nagy sarcastically asked if the quality of the content was also superior—Herbert liked Sz.-Nagy's sarcasm).

At the time of the writing of our "General Orthogonal Polynomials" Herbert did not use electronic mail yet, so we communicated via fax and ordinary mail, which was pretty slow, but he also visited us in Szeged where all my family fell in love with his charming character. I remember one rainy day walking around the national pantheon in Szeged trying to deal with some unchartered areas. Herbert stated a theorem in an arcade shading us from the rain, and I gave a counterexample in the next one. Then he formulated a different version at the next arcade, to which I gave a different counterexample, and so on. By the time our long walk was over we had Theorem 3.3.2 in our book. As everybody, Herbert also made mistakes. Once he had two rather complicated arguments for the two parts of an if and only if statement that were both proving the same direction. On another occasion, he sent me a proof using Cauchy's inequality in the wrong way. When I reminded him that the inequality goes in the opposite direction, he sent me a different proof using the correct inequality. What I want to emphasize is that Herbert saw things differently than most of us do, and even though minor technicalities might not have worked right away, his vision was correct, and eventually endured the scrutiny of mathematical rigor. He was seeing things geometrically, and in discussion he used non-mathematical terms like "potential sheet", "measure holding down polynomials", or "making zero surgery".

---

[12] http://www.math.purdue.edu/~branges/apology.pdf

He visited us in Hungary and the US several times, and on a few occasions he even stayed with me in Tampa for a couple of weeks. Besides discussing mathematics we were running together (he was a much better runner than I was), visited many beaches, went deep sea fishing, made many hikes, had numerous dinners. A few times we gave to one another long rides to our destination depending who had a car (he was traveling by car wherever it was possible, even from Berlin to Erice, Sicily). Herbert was quite talkative about almost anything except for his personal life. Even though I spent with him extended periods, I knew very little of his family and his private matters; I did not even know his age. One summer we had an extremely pleasant hike in the Alps and I repeatedly tried to find out from him his age without much success. Then, on a very long stony ski slope, Herbert casually mentioned that vipers tend to hide under sun lit stones—that was the end of my enjoyment and my quest about his age;[13] from then on I wanted to get off the slope as quickly as possible. Even though later our interest diverged, his visits were always intense with mathematical discussions (sometimes when he became excited with the problem he started talking in German; then I knew he was really concentrating). Everybody in my family loved him and always enjoyed our meetings at various places (Erice, Nice, Maratea, Oberwolfach, Budapest, and so forth). For two decades when I was going to attend a conference and saw Stahl's name among the participants, I could be sure that I would have a great time at that meeting. In January of 2011 we had a particularly memorable *Research in Pairs* workshop in Oberwolfach with Ed Saff and Nikos Stylianopoulos. Herbert was still healthy and I will always remember that two weeks of exciting mathematical debates, hikes, wonderful dinners and wines. In May of 2012 we dedicated to him a workshop in Szeged. The timing was almost perfect, he was in relatively good health and in good spirits. Many of his friends, for whom the name "Herbert" means only one thing, had the last chance to enjoy his company.

Fig. 8. One of Herbert Stahl's last lectures at 09:40–10:15 on Monday, May 28, 2012 in Szeged, Hungary.

I was talking with him several times during his illness. He knew what was coming, and spoke frankly about it (and about doctors not willing to help to shorten the final stage). These are

---

[13] In 2007, PaulN figured out Herbert's birthday from a grant application that he found on an obscure web page and this impressed VT very much.

his own words: "Compared to my parents, my life was excellent. Excellent. Just the last one hundred meters...". One never knows in such a situation if her/his call is inconvenient or not, but he assured me "...you never bother me, call me any time". But he was not answering my calls after that...

Walter Van Assche. I am not sure when I met Herbert Stahl the first time: it was probably in 1986 in Tampa at one of the approximator meetings that Ed Saff was running. Herbert already had a reputation of being one of the strongest mathematicians in complex constructive approximation and his papers on orthogonal polynomials with complex-valued weight functions [20,21] were considered as extraordinary beautiful publications. Herbert certainly liked to travel and was a 'frequent participant' at various conferences on rational approximation, orthogonal polynomials or approximation theory. He was also the perfect gentleman, willing to share his ideas with others and encouraging other researchers to work out their ideas, with good advise and with some ideas of his own.

In the 1990's we had a number of INTAS[14] projects and Herbert was a very crucial participant of these projects, building the bridge between teams from the European Union (Spain, France, Belgium, Portugal and Hungary) and from the former Soviet Union (Moscow, Nizhny Novgorod, Kharkov). He had very good contacts with researchers such as A.A. Gonchar, A.I. Aptekarev, S.P. Suetin, V.A. Kalyagin, L.B. Golinskii, and was also aware of the research activities in the European Union, in particular in Leuven, Lille, Madrid and Szeged. One of the work packages in the project NeCCA (Network on Constructive Complex Approximation) was the Riemann-Hilbert approach for Hermite-Padé approximation. Herbert had an approach for investigating the asymptotic behavior of the type I quadratic Hermite-Padé approximants to the exponential function which was based on an integral representation and analysis of a Riemann surface with three sheets. He did not publish this right away (the full result is in [77] and [79]) but he gave some glimpses in [74], [80]. Arno Kuijlaars, Franck Wielonsky and I had a different approach based on the Riemann-Hilbert problem for this Hermite-Padé problem, but it also used the Riemann surface that Herbert had found. So we decided to join forces and publish a research announcement (in French) in the Comptes Rendus Mathématique [75]. This not only gave me Stahl number 1 but was the only paper that Herbert has in French. Later we realized that our analysis not only gave the asymptotic behavior for the quadratic Hermite-Padé approximants to the exponential function, but also the asymptotics (and in particular the distribution of the zeros) of the type II Hermite-Padé approximant, i.e., the simultaneous Padé approximant with common denominator to the exponential functions e^x and e^{-x} [82]. This research activity was a good illustration that Herbert was a crucial component in linking together the ideas and techniques which were known in the research teams of the project.

Others have already reported of Herbert's interest in classical music. I have witnessed this once in Saint Petersburg at a meeting in the Euler Institute in 1994, where Herbert was very handy at finding out what was going on at the Philharmonia and bought tickets for all interested participants. The last time I met Herbert was in July 2012 when he attended a workshop in Leuven, which was a two day meeting right after a similar meeting in Lille. As usual Herbert gave a beautiful talk, not any longer in his elegant handwriting with carefully hand drawn pictures, but with a beamer presentation with again very nice pictures but generated with contemporary software.

---

[14] International Association for the Promotion of Cooperation with Scientists from the Independent States of the Former Soviet Union
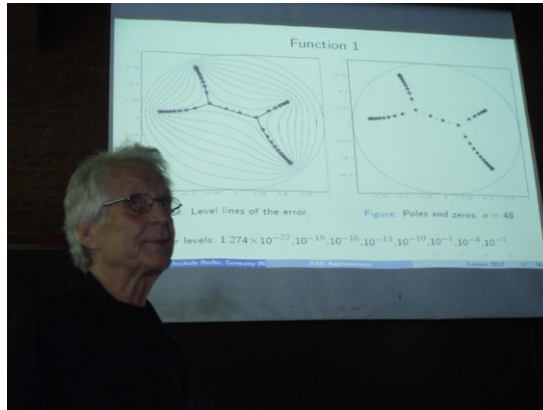
Fig. 9. Probably Herbert Stahl's very last lecture at 11:30–12:30 on Tuesday, July 3, 2012 in Leuven, Belgium.

The message was still the same: complex rational approximation is a wonderful research topic which requires deep knowledge of complex function theory, logarithmic potentials, Riemann surfaces and approximation theory, and Herbert was a master in playing with the ingredients of those areas.

Franck Wielonsky. I first met Herbert at the International Conference on Rational Approximation organized by Annie Cuyt in Antwerp in 1993. I learned from him many mathematical ideas and theorems, but I also owe to him such different experiences as sightseeing Berlin from a Mercedes, jogging on the Teufelsberg, visiting typical cafés near the Alexanderplatz, making a trip to Caputh, the place where Einstein used to do sailing during his years in Berlin, and attending opera at the Bolshoi Theater in Moscow. I will keep the remembering of a very kind and generous person, always willing to share ideas and good moments with others.

References

[REF1] D. Bessis, P. Moussa, and M. Villani, Monotonic converging variational approximations to the functional integrals in quantum statistical mechanics, J. Math. Phys., 16 (1975), pp. 2318–2325.

[REF2] D.J. Newman, Rational approximation to $|x|$, Michigan Math. J., 11 (1964), pp. 11–14.

[REF3] J. Nuttall, The convergence of Padé approximants to functions with branch points, in *Padé and rational approximation* (Proceedings of an International Symposium held at the University of South Florida, Tampa, Florida, December 15–17, 1976), E.B. Saff and R.S. Varga, eds., Academic Press, New York, 1977, pp. 101–109.

List of coauthors of Herbert Stahl

Alexander I. Aptekarev, Laurent Baratchart, Thomas Bloom, Kathy A. Driver, Michael Eiermann, Ursula-Christina Fellberg, Douglas P. Hardin, Arno B. J. Kuijlaars, Guillermo López Lagomasino, Doron S. Lubinsky, Karl-Heinz Neumann, Klaus Page, Alexander A. Pekarskiǐ, Edward B. Saff, Rainer Schlittgen, Thomas Schmelzer, Ulrich Stark, Nikos S. Stylianopoulos, Vilmos Totik, Walter Van Assche, Mikael Weigelt, Götz Wiegand, Franck Wielonsky, and Maxim L. Yattselev.

List of publications of Herbert Stahl

[HS1]   Herbert R. Stahl. "Beiträge zum Problem der Konvergenz von Padé Approximierenden (Contributions to the convergence problem of Padé approximants)." PhD thesis, TU-Berlin, 1976 (in German; a summary is contained in [19]).

[HS2]   Klaus Page, Rainer Schlittgen, and Herbert R. Stahl. Exact tests for Markov chains, in "COMPSTAT 1976. Proceedings in computational statistics. 2nd symposium held in Berlin (West)", Johannes Gordesch and Peter Naeve, eds., Physica-Verlag, Wien, 1976, pp. 46–54.

[HS3]   Klaus Page, Rainer Schlittgen, and Herbert R. Stahl. Statistische Analyse von endlichen Markoffketten, in "Probleme der Parameterschätzung bei Stochastischen Prozessen", Petre Tautu, ed., Deutsches Krebsforschungszentrum, Heidelberg, 1977, pp. 1–16 (in German).

[HS4]   Herbert R. Stahl. Orthogonal polynomials of complex-valued measures and the convergence of Padé approximants, in "Fourier analysis and approximation theory. Vol. II. Proceedings of a Colloquium held in Budapest, August 16–21, 1976", György Alexits and Pál Turán, eds., volume 19 of Colloquia Mathematica Societatis János Bolyai, North-Holland Publishing Co., Amsterdam, 1978, pp. 771–788.

[HS5]   Herbert R. Stahl. Die Klassifikation der Bevölkerungsbewegung nach Wanderungsmotiven, in "Demographische Planungsinformationen", Eckart Elsner, ed., Kulturbuch-Verlag, Berlin, 1979, pp. 283–305 (in German).

[HS6]   Ursula-Christina Fellberg, Karl-Heinz Neumann, and Herbert R. Stahl. Typische Konstellationen der Beschäftigung ausländischer Arbeitnehmer: Ergebnisse einer Cluster-Analyse von Betrieben des verarbeitenden Gewerbes (Typical employment constellations of foreign workers: results of a cluster analysis of enterprises in the manufacturing industry). Mitteilungen aus der Arbeitsmarkt- und Berufsforschung, 13:2 (1980), pp. 272–292 (in German).

[HS7]   Herbert R. Stahl. Beschreibung der Sozialstruktur in Berlin (West) mit Hilfe der Faktoranalyse. Berliner Statistik. Monatsschrift, 34:3 (1980), pp. 37–52 (in German).

[HS8]   Herbert R. Stahl. On the convergence of Padé approximants to Hamburger functions, in "Constructive theory of functions. Proceedings of the international conference, Bulgaria, Blagoevgrad, 1977", Blagovest Kh. Sendov and Dimitir K. Vachov, eds., Publishing House of the Bulgarian Academy of Sciences Sofia, 1980, pp. 497–500.

[HS9]   Herbert R. Stahl. "Eine Methode zur graphischen Darstellung multivariater Daten (A method for graphical representation of multivariate data)." PhD thesis, TU-Berlin, 1981 (in German).

[HS10] Herbert R. Stahl. "Clusteranalyse großer Objektmengen mit problemorientierten Distanzmaßen (Cluster analysis of large quantities of objects with problem-oriented distance measurements)." Habilitation thesis, TU-Berlin, Berlin, 1983 (in German).

[HS11] Herbert R. Stahl. Contribution to "Lexikon der Informatik und Datenverarbeitung", Hans-Jochen Schneider, ed., Oldenbourg, München, 1983 (in German).

[HS12] Herbert R. Stahl. On the divergence of certain Padé approximant and the behaviour of the associated orthogonal polynomials, in "Orthogonal polynomials and applications. Proceedings of the Laguerre Symposium held at Bar-le-Duc, October 15–18, 1984", Claude Brezinski, André Draux, Alphonse P. Magnus, Pascal Maroni, and André Ronveaux, eds., volume 1171 of Lecture Notes in Math., Springer-Verlag, Berlin, 1985, pp. 321–330.

[HS13] Herbert R. Stahl. "Clusteranalyse großer Objektmengen mit problemorientierten Distanzmaßen (Cluster analysis of large quantities of objects with problem-oriented distance measurements)", volume 338 of Reihe Wirtschaftswissenschaften. Thun, H. Deutsch, Frankfurt am Main, 1985 (in German).

[HS14] Herbert R. Stahl. Divergence of diagonal Padé approximants and the asymptotic behavior of orthogonal polynomials associated with nonpositive measures. Constr. Approx., 1:3 (1985), pp. 249–270.

[HS15] Herbert R. Stahl. Extremal domains associated with an analytic function. I. Complex Variables Theory Appl., 4:4 (1985), pp. 311–324.

[HS16] Herbert R. Stahl. Extremal domains associated with an analytic function. II. Complex Variables Theory Appl., 4:4 (1985), pp. 325–338.

[HS17] Herbert R. Stahl. The structure of extremal domains associated with an analytic function. Complex Variables Theory Appl., 4:4 (1985), pp. 339–354.

[HS18] Herbert R. Stahl. Cluster analysis of large data sets, in "Classification as a tool of research. Proceedings of the ninth annual meeting of the German Classification Society held at the University of Karlsruhe, Germany, Karlsruhe, June 26–28, 1985", Wolfgang Gaul and Martin Schader, eds., North-Holland Publishing Co., Amsterdam, 1986, pp. 423–430.

[HS19] Herbert R. Stahl. Non-diagonal Padé approximants to Markov functions, in "Approximation theory V. Proceedings of the fifth international symposium held at Texas A & M University, College Station, Texas, January 13–17, 1986", Charles K. Chui, Larry L. Schumaker, and Joseph D. Ward, eds., Academic Press Inc., Boston, MA, 1986, pp. 571–574.

[HS20] Herbert R. Stahl. Orthogonal polynomials with complex-valued weight function. I. Constr. Approx., 2:3 (1986), pp. 225–240.

[HS21] Herbert R. Stahl. Orthogonal polynomials with complex-valued weight function. II. Constr. Approx., 2:3 (1986), pp. 241–251.

[HS22] Herbert R. Stahl. A note on a theorem by H. N. Mhaskar and E. B. Saff: "Where does the sup norm of a weighted polynomial live? (A generalization of incomplete polynomials)" [Constr. Approx. 1:1 (1985), pp. 71–91], in "Approximation theory, Tampa. Proceedings of the seminar held in Tampa, Fla., 1985–1986", Edward B. Saff, ed., volume 1287 of Lecture Notes in Math., Springer-Verlag, Berlin, 1987, pp. 176–179.

[HS23] Herbert R. Stahl. Existence and uniqueness of rational interpolants with free and prescribed poles, in "Approximation theory, Tampa. Proceedings of the seminar held in Tampa, Fla., 1985–1986", Edward B. Saff, ed., volume 1287 of Lecture Notes in Math., Springer-Verlag, Berlin, 1987, pp. 180–208.

[HS24] Herbert R. Stahl. A note on three conjectures by Gonchar on rational approximation. J. Approx. Theory, 50:1 (1987), pp. 3–7.

[HS25] Herbert R. Stahl. Three different approaches to a proof of convergence for Padé approximants, in "Rational approximation and applications in mathematics and physics. Proceedings of the Łańcut conference held in Łańcut, 1985", Jacek Gilewicz, Maciej Pindor, and Wojciech Siemaszko, eds., volume 1237 of Lecture Notes in Math., Springer-Verlag, Berlin, 1987, pp. 79–124.

[HS26] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. Box-Jenkins-Analyse der Monatsmittelwerte von Immissionsmessungen verschiedener Schadstoffe an unterschiedlichen Stationen. Monatsberichte aus dem Meßnetz des Umweltbundesamtes Berlin, 7/86 (1987), pp. 5–41 (in German).

[HS27] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. Multivariate Regressionsanalyse der Tagesmittelwerte von Immissionsmessungen ohne Berücksichtigung der Zeitreiheneigenschaft. Monatsberichte aus dem Meßnetz des Umweltbundesamtes Berlin, 11/86 (1987), pp. 4–41 (in German).

[HS28] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. Box-Jenkins-Analyse der Tagesmittelwerte von Immissionsmessungen verschiedener Schadstoffe an unterschiedlichen Stationen. Monatsberichte aus dem Meßnetz des Umweltbundesamtes Berlin, 12/86 (1987) pp. 5–36 (in German).

[HS29] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. Transfer-Funktionsmodelle der Tagesmittelwerte von fünf Immissions Zeitreihen. Monatsberichte aus dem Meßnetz des Umweltbundesamtes Berlin, 2/87 (1987), pp. 33–72 (in German).

[HS30] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. Box-Jenkins analysis of the monthly averages for concentration measurements of various air pollutants at different stations, in "Proceedings of the EMEP Workshop on Data Analysis and Presentation, Cologne, FRG, June 15–17, 1987", volume 7 of EMEP/CCC-Report, Norsk Institutt for Luftforskning, Lillestrøm, December 1987, pp. 237–262.

[HS31] Götz Wiegand, Mikael Weigelt, and Herbert R. Stahl. "Methoden zur Bewertung der Luftqualität. Teil A: Zeitreihenanalyse von Immissionsmessungen." Umweltforschungsplan des Bundesministers für Umwelt, Naturschutz und Reaktorsicherheit, GEOMET — Umweltberatung und Regionalplanung GmbH, Berlin, 1987 (in German).

[HS32] Herbert R. Stahl, Mikael Weigelt, and Götz Wiegand. Box-Jenkins Analysis of Air Pollution Data, in "Papers of the 16th Annual Meeting of DGOR in Cooperation with NSOR / Vorträge der 16. Jahrestagung der DGOR zusammen mit der NSOR (Veldhoven, Netherlands, September 23–25, 1987)", Helmut Schellhaas, Paul van Beek, Heinz Isermann, Reinhart Schmidt, Mynt Zijlstra, eds., volume 1987 of Operations Research Proceedings, Springer-Verlag, Berlin etc., 1988, pp. 417–424.

[HS33] Herbert R. Stahl. Asymptotics of Hermite-Padé polynomials and related convergence results—a summary of results, in "Nonlinear numerical methods and rational approximation. Proceedings of the conference held at the University of Antwerp, Wilrijk, April 20–24, 1987", Annie A. M. Cuyt, ed., volume 43 of Math. Appl., Reidel, Dordrecht, 1988, pp. 23–53.

[HS34] Herbert R. Stahl. nth-root asymptotics of orthonormal polynomials and nondiagonal Padé approximants, in "Orthogonal polynomials and their applications. Proceedings of the Second International Symposium held in Segovia, September 22–27, 1986", Manuel Alfaro, Jesús S. Dehesa, Francisco Marcellán Español, José L. Rubio de Francia, and Jaime Vinuesa Tejedor, eds., volume 1329 of Lecture Notes in Math., Springer-Verlag, Berlin, 1988, pp. 308–319.

[HS35] Herbert R. Stahl. General convergence results for rational approximants, in "Approximation theory VI, Vol. II. Proceedings of the Sixth International Symposium held at Texas A & M University, College Station, Texas, January 6–10, 1989", Charles K. Chui, Larry L. Schumaker, and Joseph D. Ward, eds., Academic Press, Boston, MA, 1989, pp. 605–634.

[HS36] Herbert R. Stahl. On the convergence of generalized Padé approximants. Constr. Approx., 5:2 (1989), pp. 221–240.

[HS37] Herbert R. Stahl and Vilmos Totik. *n*th root asymptotic behavior of orthonormal polynomials, in "Orthogonal polynomials. Theory and practice. Proceedings of the NATO Advanced Study Institute on Orthogonal Polynomials and their Applications held at The Ohio State University, Columbus, Ohio, May 22–June 3, 1989", edited by Paul Nevai with the assistance of Mourad E. H. Ismail, volume 294 of NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., Kluwer Acad. Publ., Dordrecht, 1990, pp. 395–417.

[HS38] Herbert R. Stahl. Orthogonal polynomials with respect to complex-valued measures, in "Orthogonal polynomials and their applications. Proceedings of the Third International Symposium held in Erice, June 1–8, 1990", Claude Brezinski, Laura Gori, and André Ronveaux, eds., volume 9 of IMACS Ann. Comput. Appl. Math., Baltzer, Basel, 1991, pp. 139–154.

[HS39] Herbert R. Stahl. Best rational approximants to Markov functions, in "Approximation theory. Papers from the conference held in Kecskemét, August 6–11, 1990", József Szabados and Károly Tandori, eds., volume 58 of Colloquia Mathematica Societatis János Bolyai, North-Holland Publishing Co, Amsterdam, 1991, pp. 627–643.

[HS40] Herbert R. Stahl. On regular *n*th root asymptotic behavior of orthonormal polynomials. J. Approx. Theory, 66:2 (1991), pp. 125–161.

[HS41] Alexander I. Aptekarev and Herbert R. Stahl. Asymptotics of Hermite-Padé polynomials, in "Progress in approximation theory. An international perspective. Proceedings of the International Conference on Approximation Theory held at the University of South Florida, Tampa, Fla., March 19–22, 1990", Andrei A. Gonchar and Edward B. Saff, eds., volume 19 of Springer Ser. Comput. Math., Springer-Verlag, New York, 1992, pp. 127–167.

[HS42] Thomas Bloom, Doron S. Lubinsky, and Herbert R. Stahl. Interpolatory integration rules and orthogonal polynomials with varying weights, in "Extrapolation and rational approximation. Proceedings of the International Mathematical Congress held in Puerto de la Cruz, January 13–17, 1992.", Numer. Algorithms, 3:1-4 (1992), pp. 55–65.

[HS43] Herbert R. Stahl and Edward B. Saff. Sequences in the Walsh Table of $|x|^a$, in "Constructive theory of functions. Proceedings of the international conference, Bulgaria, Varna, 1991", Kamen G. Ivanov, Blagovest Kh. Sendov, and Pencho P. Petrushev, eds., Publishing House of the Bulgarian Academy of Sciences, Sofia, 1992, pp. 249–257.

[HS44] Herbert R. Stahl and Ulrich Stark. Sozialstruktur der Bevölkerung in den Statistischen Gebieten von Berlin (West) : faktoranalytische Untersuchung auf der Basis der Ergebnisse der Volkszählung 1987 (Social structure of the population in the statistical areas of West Berlin: factor-analytical study based on the results of the census in 1987). Berliner Statistik. Monatsschrift, 46:4 (1992), pp. 117–130 (in German).

[HS45] Herbert R. Stahl and Vilmos Totik. "General orthogonal polynomials", volume 43 of Encyclopedia of Mathematics and its Applications. Cambridge University Press, Cambridge, 1992.

[HS46] Herbert R. Stahl. Best uniform rational approximations of $|x|$ on $[-1, 1]$. Mat. Sb., 183:8 (1992), pp. 85–118 (in Russian). Translation in Russian Acad. Sci. Sb. Math. 76:2 (1993), pp. 461–487.

[HS47] Herbert R. Stahl. Uniform rational approximation of $|x|$, in "Methods of approximation theory in complex analysis and mathematical physics. Selected papers from the International Seminar held in Leningrad, May 13–26, 1991", Andrei A. Gonchar and Edward B. Saff, eds., volume 1550 of Lecture Notes in Math., Reissued by Springer-Verlag, Berlin, 1993, pp. 110–130.

[HS48] Thomas Bloom, Doron S. Lubinsky, and Herbert R. Stahl. Distribution of points in convergent sequences of interpolatory integration rules: the rates, in "Numerical integration, IV. Proceedings of the Fourth Conference held in Oberwolfach, November 8–14, 1992", Helmut Braß and Günther Hämmerlin, eds., volume 112 of Internat. Ser. Numer. Math., Birkhäuser, Basel, 1993, pp. 25–37.

[HS49] Thomas Bloom, Doron S. Lubinsky, and Herbert R. Stahl. What distributions of points are possible for convergent sequences of interpolatory integration rules? Constr. Approx., 9:1 (1993), pp. 41–58.

[HS50] Thomas Bloom, Doron S. Lubinsky, and Herbert R. Stahl. Distribution of points for convergent interpolatory integration rules on $(-\infty, \infty)$. Constr. Approx., 9:1 (1993), pp. 59–82.

[HS51] Herbert R. Stahl. Best uniform rational approximation of x^α on $[0, 1]$. Bull. Amer. Math. Soc. (N.S.), 28:1 (1993), pp. 116–122.

[HS52] Kathy A. Driver and Herbert R. Stahl. Normality and error formulae for simultaneous rational approximants to Nikishin systems, in "Nonlinear numerical methods and rational approximation, II. Proceedings of the International Conference held at the University of Antwerp, Wilrijk, September 5–11, 1993", Annie A. M. Cuyt, ed., volume 296 of Math. Appl., Kluwer Acad. Publ., Dordrecht, 1994, pp. 97–105.

[HS53] Kathy A. Driver and Herbert R. Stahl. Normality in Nikishin systems. Indag. Math. (N.S.), 5:2 (1994), pp. 161–187.

[HS54] Michael Eiermann and Herbert R. Stahl. Zeros of orthogonal polynomials on regular N-gons, in "Linear and Complex Analysis Problem Book 3. Part II.", Victor P. Havin and Nikolai K. Nikolski, eds., volume 1574 of Lecture Notes in Mathematics, chapter "Orthogonal polynomials", edited by Paul Nevai, Springer-Verlag, Berlin, 1994, pp. 187–189.

[HS55] Herbert R. Stahl. Poles and zeros of best rational approximants of $|x|$. Constr. Approx., 10:4 (1994), pp. 469–522.

[HS56] Edward B. Saff and Herbert R. Stahl. Asymptotic distribution of poles and zeros of best rational approximants to x^α on [0, 1], in "Topics in complex analysis. Proceedings of the Semester on Complex Analysis held in Warsaw, 1992", Piotr Jakóbczak and Wiesław Pleśniak, eds., volume 31 of Banach Center Publications, Polish Acad. Sci. Inst. Math., Warsaw, 1995, pp. 329–348.

[HS57] Kathy A. Driver and Herbert R. Stahl. Simultaneous rational approximants to Nikishin systems of two functions, in "Computational methods and function theory 1994. Papers from the Second International Conference (CMFT '94) held at the Universiti Sains Malaysia, Penang, March 21–25, 1994", Rosihan M. Ali, Stephan Ruscheweyh, and Edward B. Saff, eds., volume 5 of Ser. Approx. Decompos., World Sci. Publ., River Edge, NJ, 1995, pp. 135–146.

[HS58] Herbert R. Stahl. Simultaneous rational approximants, in "Computational methods and function theory 1994. Papers from the Second International Conference (CMFT '94) held at the Universiti Sains Malaysia, Penang, March 21–25, 1994", Rosihan M. Ali, Stephan Ruscheweyh, and Edward B. Saff, eds., volume 5 of Ser. Approx. Decompos., World Sci. Publ., River Edge, NJ, 1995, pp. 325–349.

[HS59] Kathy A. Driver and Herbert R. Stahl. Simultaneous rational approximants to Nikishin-systems. I. Acta Sci. Math. (Szeged), 60:1-2 (1995), pp. 245–263.

[HS60] Kathy A. Driver and Herbert R. Stahl. Simultaneous rational approximants to Nikishin systems. II. Acta Sci. Math. (Szeged), 61:1-4 (1995), pp. 261–284.

[HS61] Alexander A. Pekarskiĭ and Herbert R. Stahl. Bernstein-type inequalities for derivatives of rational functions in L_p spaces, $p < 1$. Mat. Sb., 186:1 (1995), pp. 119–130 (in Russian). Translation in Sb. Math. 186:1 (1995), pp. 121–131.

[HS62] Herbert R. Stahl. Convergence of rational interpolants, in "Numerical analysis. Papers from the Conference on Special Topics in Numerical Analysis and Applied Mathematics in honor of Jean Meinguet held in Louvain-la-Neuve, December 1, 1995", Bull. Belg. Math. Soc. Simon Stevin, Société Mathématique de Belgique, Brussels, 1996, pp. 11–32.

[HS63] Herbert R. Stahl. Diagonal Padé approximants to hyperelliptic functions, in "100 ans après Th.-J. Stieltjes. Papers from the meeting held at the Université Paul-Sabatier, Toulouse, March 1995", Ann. Fac. Sci. Toulouse Math. (6), Special issue, Université Paul Sabatier, Toulouse, 1996, pp. 121–193.

[HS64] Herbert R. Stahl. The convergence of diagonal Padé approximants and the Padé conjecture. J. Comput. Appl. Math., 86:1 (1997), Special issue dedicated to William B. Gragg (Monterey, CA, 1996), pp. 287–296.

[HS65] Edward B. Saff and Herbert R. Stahl. Ray sequences of best rational approximants for $|x|^\alpha$. Canad. J. Math., 49:5 (1997), pp. 1034–1065.

[HS66] Herbert R. Stahl. The convergence of Padé approximants to functions with branch points. J. Approx. Theory, 91:2 (1997), pp. 139–204.

[HS67] Herbert R. Stahl. Conjectures around the Baker-Gammel-Wills conjecture. Constr. Approx., 13:2 (1997), pp. 287–292. It was reproduced in a wrong format; the correction is printed in Constr. Approx., 13:3 (1997).

[HS68] Herbert R. Stahl. Spurious poles in Padé approximation, in "Proceedings of the VIIIth Symposium on Orthogonal Polynomials and Their Applications held at the University of Seville, Seville, September 22–26, 1997", Antonio J. Duran, ed., J. Comput. Appl. Math. 99:1-2 (1998), pp. 511–527.

[HS69] Laurent Baratchart, Herbert R. Stahl, and Franck Wielonsky. Non-uniqueness of rational best approximants, in "Continued fractions and geometric function theory (CONFUN). Proceedings of the conference held in honor of Haakon Waadeland on the occasion of his 70th birthday in Trondheim, June 24–28, 1997", J. Comput. Appl. Math., 105:1-2 (1999), pp. 141–154.

[HS70] Herbert R. Stahl. Rational best approximants in the Hardy space H^2, in "Self-similar systems. International workshop, July 30–August 7, 1998, Dubna, Russia", Vyatcheslav B. Priezzhev and Vyacheslav P. Spiridonov, eds., Joint Inst. Nuclear Res, Dubna, 1999, pp. 362–383.

[HS71] Herbert R. Stahl. Strong asymptotics for orthonormal polynomials with varying weights. Acta Sci. Math. (Szeged), 66:1-2 (2000), pp. 147–192.

[HS72] Laurent Baratchart, Herbert R. Stahl, and Franck Wielonsky. Asymptotic error estimates for L^2 best rational approximants to Markov functions. J. Approx. Theory, 108:1 (2001), pp. 53–96.

[HS73] Laurent Baratchart, Herbert R. Stahl, and Franck Wielonsky. Asymptotic uniqueness of best rational approximants of given degree to Markov functions in $\mathcal{L}^2$ of the circle. Constr. Approx., 17:1 (2001), pp. 103–138.

[HS74] Herbert R. Stahl. Asymptotics for quadratic Hermite-Padé polynomials associated with the exponential function, in "Orthogonal polynomials, approximation theory, and harmonic analysis. Selected papers from the Third International Workshop held in, Inzell, Germany, April 14–18, 2000", Electron. Trans. Numer. Anal., 14 (2002), pp. 195–222 (electronic).

[HS75] Arno B. J. Kuijlaars, Herbert R. Stahl, Walter Van Assche, and Franck Wielonsky. Asymptotique des approximants de Hermite-Padé quadratiques de la fonction exponentielle et problèmes de Riemann-Hilbert (Asymptotic behavior of quadratic Hermite-Padé approximants to the exponential function and Riemann-Hilbert problems). C. R. Math. Acad. Sci. Paris, 336:11 (2003), pp. 893–896 (in French).

[HS76] Herbert R. Stahl. Best uniform rational approximation of x^α on [0, 1]. Acta Math., 190:2 (2003), pp. 241–306.

[HS77] Herbert R. Stahl. Quadratic Hermite-Padé polynomials associated with the exponential function. J. Approx. Theory, 125:2 (2003), pp. 238–294.

[HS78] Doron S. Lubinsky and Herbert R. Stahl. Some explicit biorthogonal polynomials, in "Approximation theory XI: Gatlinburg 2004. Proceedings of the International Conference held in Gatlinburg, TN, May 18–22, 2004", Charles K. Chui, Marian Neamtu, and Larry L. Schumaker, eds., Modern Methods in Mathematics, Nashboro Press, Brentwood, TN, 2005, pp. 279–285.

[HS79] Herbert R. Stahl. Asymptotic distributions of zeros of quadratic Hermite-Padé polynomials associated with the exponential function. Constr. Approx., 23:2 (2006), pp. 121–164.

[HS80] Herbert R. Stahl. From Taylor to quadratic Hermite-Padé polynomials. Electron. Trans. Numer. Anal., 25 (2006), pp. 480–510 (electronic).

[HS81] Kathy A. Driver and Herbert R. Stahl. The uniqueness of an orthogonality measure. J. Comput. Appl. Math., 207:2 (2007), pp. 180–185.

[HS82] Arno B. J. Kuijlaars, Herbert R. Stahl, Walter Van Assche, and Franck Wielonsky. Type II Hermite-Padé approximation to the exponential function. J. Comput. Appl. Math., 207:2 (2007), pp. 227–244.

[HS83] Douglas P. Hardin, Edward B. Saff, and Herbert R. Stahl. Support of the logarithmic equilibrium measure on sets of revolution in $\mathbb{R}^3$. J. Math. Phys., 48:2 (2007), 022901, 14 pp.

[HS84] Herbert R. Stahl and Thomas Schmelzer. An extension of the '1/9'-problem. J. Comput. Appl. Math., 233:3 (2009), pp. 821–834.

[HS85] Herbert R. Stahl. A potential-theoretic problem connected with complex orthogonality, in "Recent trends in orthogonal polynomials and approximation theory. Papers from the International Workshop on Orthogonal Polynomials and Approximation Theory in honor of Guillermo López Lagomasino's 60th birthday held at the Universidad Carlos III de Madrid, Leganés, September 8–12, 2008", Jorge Arvesú, Francisco Marcellán Español, and Andrei Martínez-Finkelshtein, eds., volume 507 of Contemporary Mathematics, Amer. Math. Soc., Providence, RI, 2010, pp. 255–285.

[HS86] Alexander I. Aptekarev, Guillermo López Lagomasino, Edward B. Saff, Vilmos Totik, and Herbert R. Stahl. Andreĭ Aleksandrovich Gonchar (on the occasion of his eightieth birthday). Uspekhi Mat. Nauk, 66:6(402) (2011), pp. 197–204 (in Russian). Translation in Russian Math. Surveys 66:6 (2011), pp. 1209–1216.

[HS87] Laurent Baratchart, Herbert R. Stahl, and Maxim L. Yattselev. Weighted extremal domains and best rational approximation. Adv. Math., 229:1 (2012), pp. 357–407.

[HS88] Herbert R. Stahl. Sets of Minimal Capacity and Extremal Domains. Preprint, 2012, available online from http://arxiv.org/abs/1205.3811.

[HS89] Herbert R. Stahl. Proof of the BMV Conjecture. Acta Math., 211:2 (2013), pp. 255–290.

[HS90] Doron S. Lubinsky, Avram Sidi, and Herbert Stahl. Asymptotic zero distribution of biorthogonal polynomials. J. Approx. Theory, (to be published in the volume dedicated to A.A. Gonchar and H. Stahl)

[HS91] Edward B. Saff, Herbert R. Stahl, Nikos S. Stylianopoulos, and Vilmos Totik. Bergman orthogonal polynomials on an archipelago having lakes, with applications to image recovery. (to be submitted)

[HS92] Laurent Baratchart, Herbert R. Stahl, and Maxim Yattselev. Asymptotics of AAK approximants to functions with branchpoints. (to be submitted)

Acknowledgments

Andrei Martínez-Finkelshtein's camera, #8 by Andrei, #3 by Diego Dominici, and #9 by Walter Van Assche.

Alexander Aptekarev

Keldysh Institute of Applied Mathematics, Moscow, Russia
E-mail address: aptekaa@keldysh.ru.

Paul Nevai *

KAU, Jeddah, Saudi Arabia
Upper Arlington (Columbus), Ohio, USA
E-mail address: paul@nevai.us.

Vilmos Totik

Bolyai Institute, Analysis Research Group of the Hungarian Academy of Sciences, University of
Szeged, Szeged, Aradi v. tere 1, 6720, Hungary
Department of Mathematics, University of South Florida, 4202 E. Fowler Ave, CMC342,
Tampa, FL 33620-5700, USA
E-mail address: totik@mail.usf.edu.

* Corresponding author.