

VLADIMIR VASILIEVICH ZHUK

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A SHORT SURVEY OF MATHEMATICAL WORKS

M.A. SKOPINA, O.L. VINOGRADOV

Vladimir Vasilievich Zhuk graduated from Leningrad University in 1962 under the scientific supervision of I. P. Natanson. He defended his Ph.D. thesis in 1965 that was under the supervision of G. I. Natanson, and his D.Sc. degree was obtained in 1994. For most of his life Zhuk worked in Leningrad, later Saint Petersburg, University. He was a permanent and active participant of the seminar “Constructive Function Theory” organized by I. P. and G. I. Natanson.

Zhuk’s main scientific interests lay at the very core of approximation theory, namely, in approximation of periodic functions by trigonometric polynomials: direct and inverse theorems, moduli of smoothness (called by him higher-order moduli of continuity), linear approximation methods, saturation classes, sharp constants and strong approximation. He dealt a lot with various topics such as approximation on a segment and on the real line, approximation by algebraic polynomials, splines, entire functions of exponential type and shifts of a single function, as well as approximation in abstract Banach spaces and more. However, Zhuk mostly considered these extensions as implementation of general principles that firstly needed to be understood in the most classical periodic case.

Zhuk published his first papers in 1965. From the 1980’s Zhuk published some of his results, including the new ones (not submitted to a journal) first in books, in particular, in the monographs [1,4] and textbooks [3,5–7]. These books were never translated from Russian. Some of Zhuk’s early theorems were included in the monograph [1], which became a reference book for many approximation people, especially in Leningrad. We point out two of Zhuk’s remarkable results from that period:

- 1) Zhuk determined exact orders of approximation by linear methods (in particular, by the Fejér method) for each individual function in terms of moduli of smoothness;

- 2) Zhuk found the sharp constant in the uniform Jackson inequality for the first modulus of the first derivative.

In the middle of the 1980's Zhuk focused his attention on strong approximation. During this period he wrote the monograph [4], which gave a completely new point of view on the topic. As an example, we mention that Zhuk described the Lipschitz class in terms of strong approximation.

About 20 papers Zhuk wrote are in collaboration with G. I. Natanson. Their first joint article was published in 1969, and the last joint paper was submitted to the special volume dedicated to G. I.'s 70th birthday, which was published in 2004 after he passed away. In addition to approximation of periodic functions by trigonometric polynomials, the joint works of Zhuk and G. I. Natanson affected approximation on segments, on simplexes, in abstract Banach spaces, as well as spline approximation. That year, splines became more and more popular, and the authors obtained several direct and inverse theorems of spline approximation. In addition Zhuk and G. I. Natanson wrote the textbook [2] where elements of modern approximation theory, presented in a form suitable not only for pure mathematicians, were included.

The book [6] consists of two fully independent parts written by the authors separately. The first part "Approximation of Functions" belongs to Zhuk. The book is called a textbook, but according to the content this part is rather a monograph, and it contains original results. On the other hand, the book [7] is really a textbook. It contains material on Fourier series and approximation, presented in a number of lectures; the first part is aimed at the general course of analysis and the second part at a special course on approximation theory. As usual, some original results are included.

Zhuk wrote more than 20 papers in collaboration with O. L. Vinogradov during the period of 1995–2013. Most of their joint work dealt with extremal problems of approximation theory. In particular, they thoroughly investigated the behaviour of the constants in the Jackson inequalities when the order of the modulus of continuity increases.

In the 2000's Zhuk wrote many papers jointly with his colleagues and students, among them we mention N. Yu. Dodonov, G. Yu. Puerov, O. A. Tumka, and M. V. Babushkin.

Zhuk was very interested in the history of approximation theory. He is an author or an editor of several review papers and books devoted to mathematicians. The papers [9–14], in particular, are related to approximation theory.

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